

Online Detection of Repetitions with Backtracking

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Repetitions

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- ▶ $\Omega(n \log n)$ unordered alphabet [Main, Lorentz 85]
- ▶ $\Theta(n \log n)$ $\epsilon=2$, unordered alphabet [Apostolico, Breslauer 96]
- ▶ $O(n \log \sigma)$ ordered alphabet [Hong, Chen 08]
(σ is the alphabet size)

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Theorem 1

For unordered alphabet, there is an ϵ -repetition detector working in $O(n \log m)$ time and $O(m)$ space, where m is the length of a longest string generated by a given sequence of n *append* and *backtrack* operations.

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For unordered alphabet, there is an ϵ -repetition detector working in $O(n \log m)$ time and $O(m)$ space, where m is the length of a longest string generated by a given sequence of n *append* and *backtrack* operations.

Theorem 2 (our method differs from [Hong, Chen 08])

For ordered alphabet, there is an ϵ -repetition detector without backtracking working in $O(n \log \sigma)$ time and $O(n)$ space, where n is the number of *append* operations and σ is the alphabet size.

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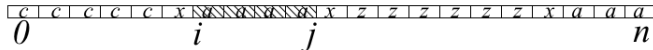
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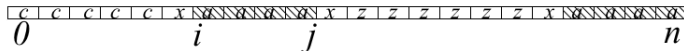


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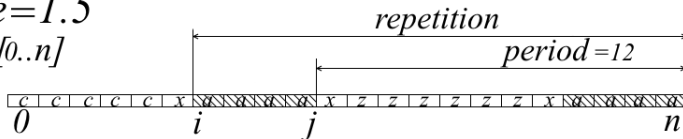
append(a)



Catcher

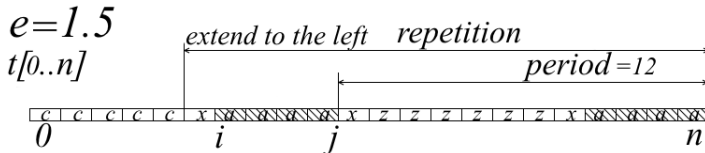
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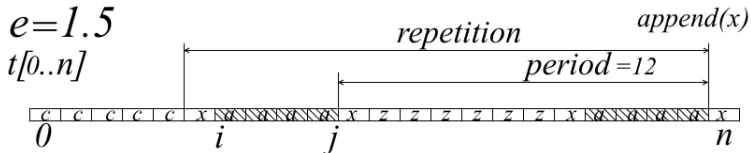
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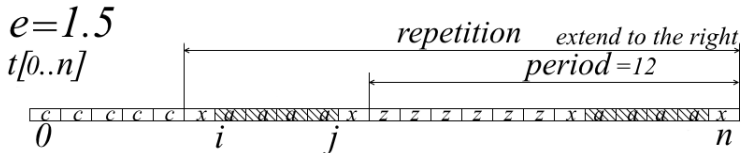
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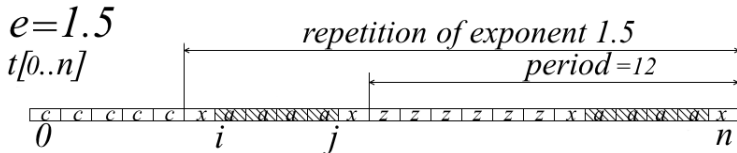
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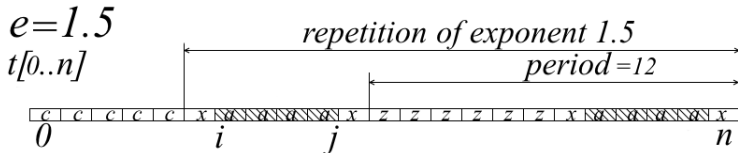
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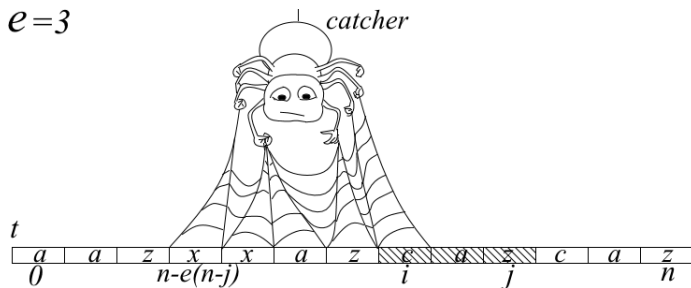
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Lemma

If $t[k..n]$ is an e -repetition and $n - e(n-j) \leq k \leq i$, the catcher finds this repetition.

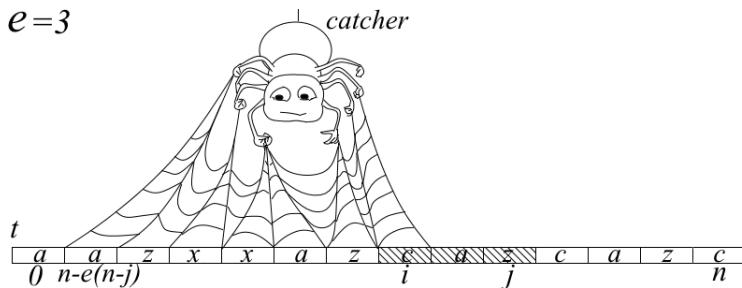
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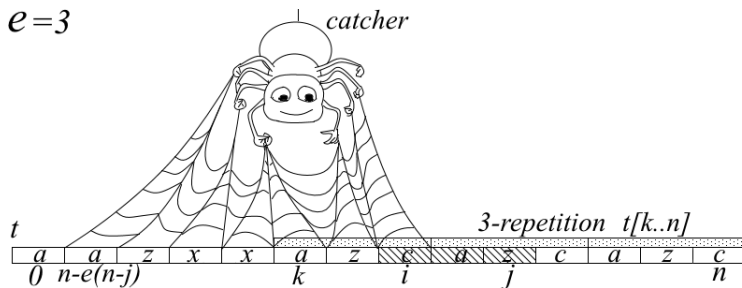
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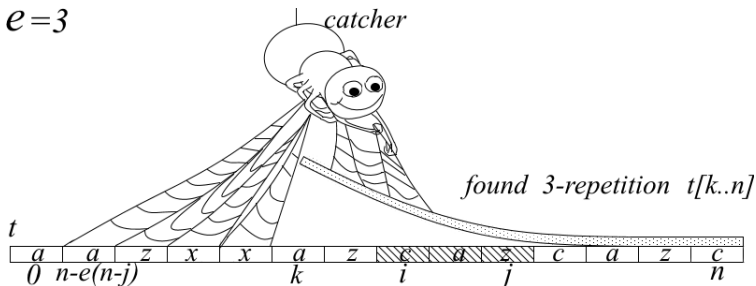


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$e=3$



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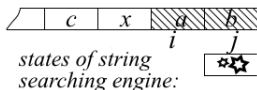
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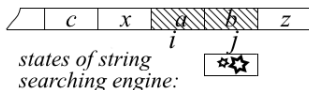
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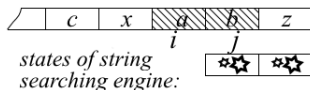
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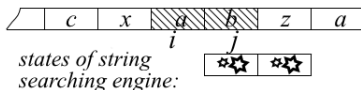
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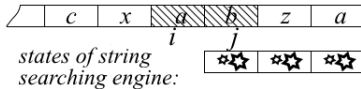
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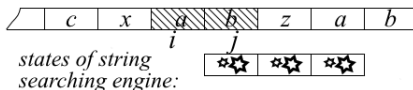
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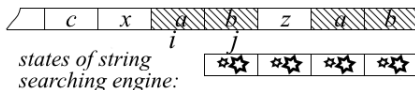
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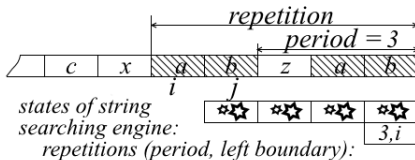
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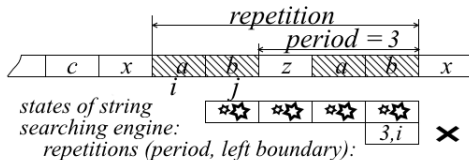
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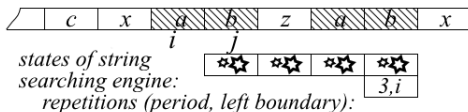
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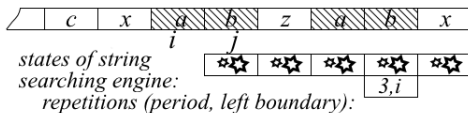
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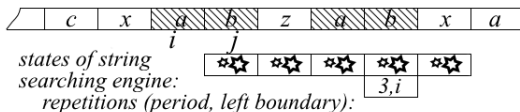
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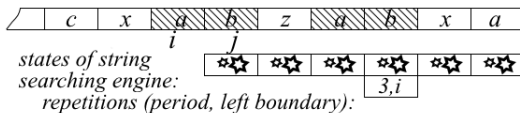
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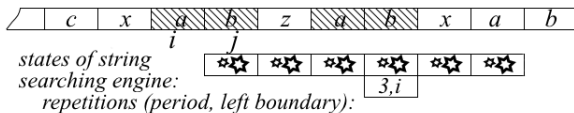
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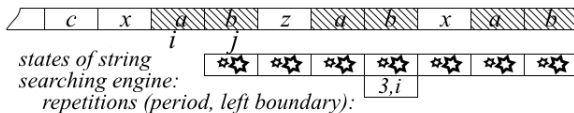
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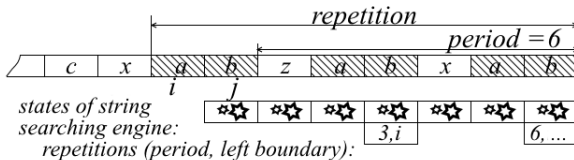
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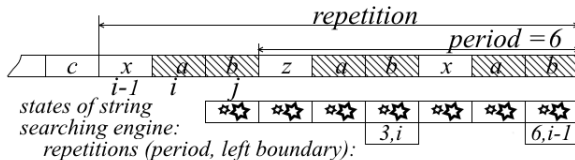
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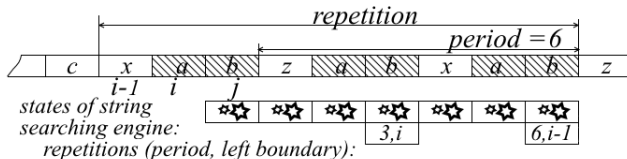
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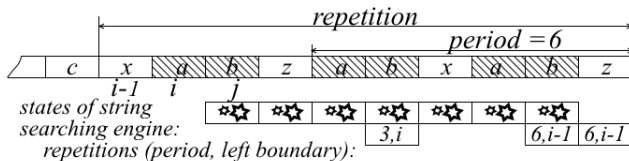
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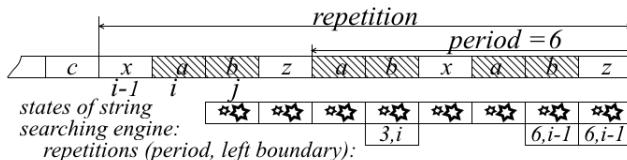
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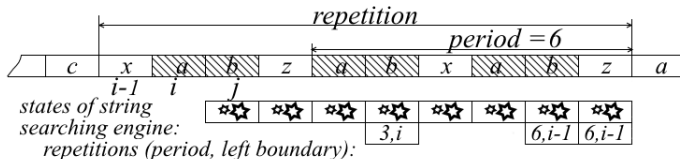
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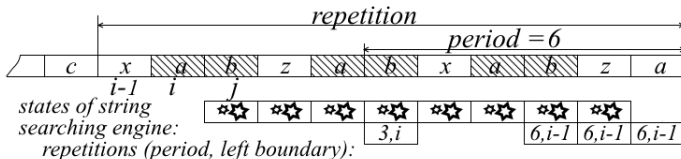
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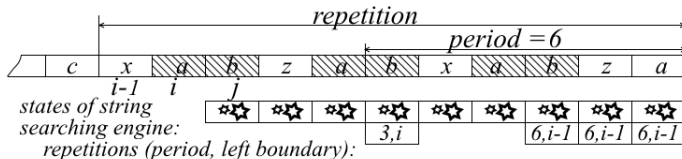
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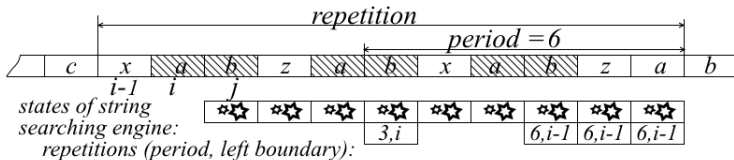
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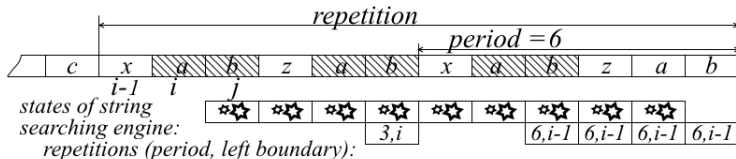
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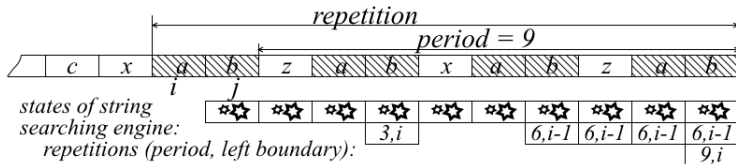
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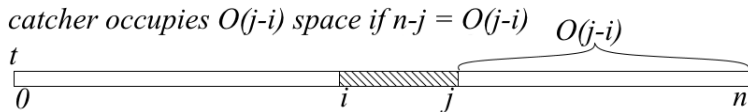
Lemma

Fix $c > 0$. If $t[0..|t| - 2]$ is e -repetition-free and $c(j - i + 1) \geq n - i$, then the states occupy $O((c + 1)(n - i))$ space.

Time and space consumptions

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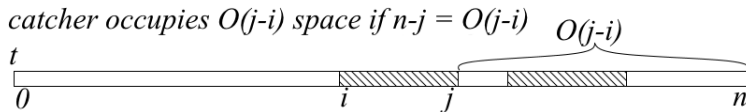
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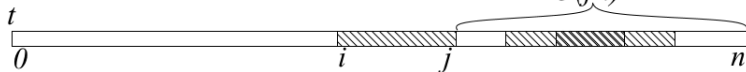


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catcher occupies $O(j-i)$ space if $n-j = O(j-i)$

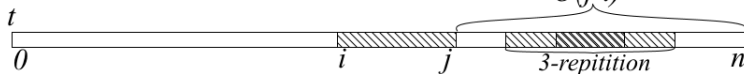


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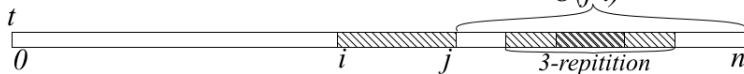


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Performing left extensions lazily, we can achieve $O(c)$ time for each modification of the catcher (see details in the paper)

Idea of the detector with backtracking

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Maintain $O(\log m)$ catchers “covering” the string $t[0..m]$

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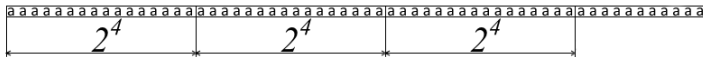
[illegible]

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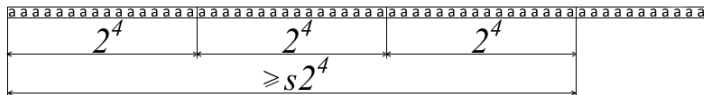


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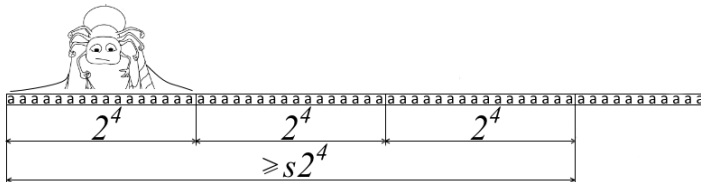


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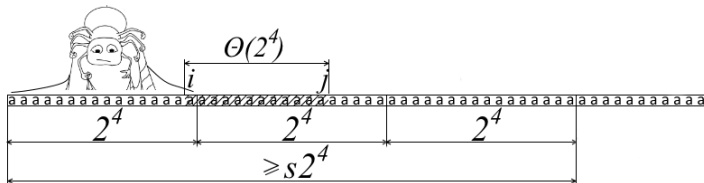


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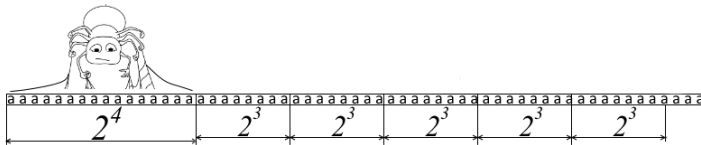


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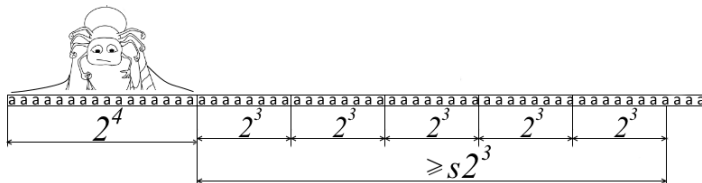


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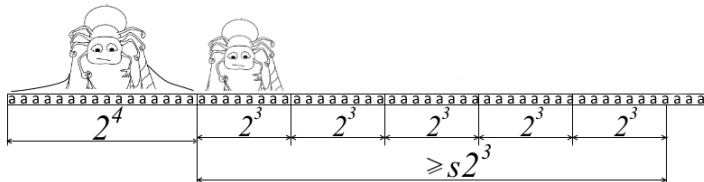


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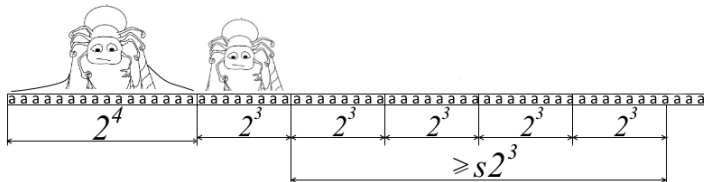


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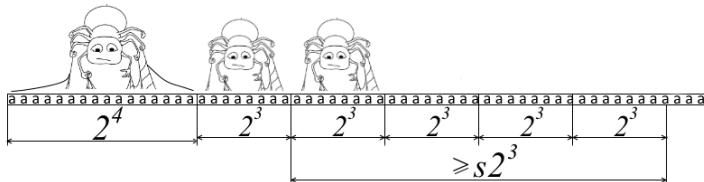


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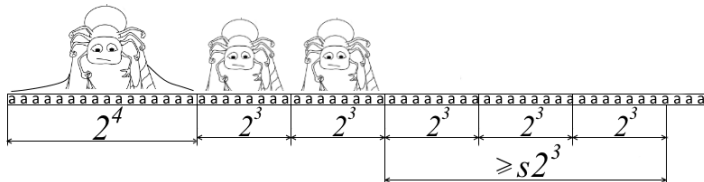


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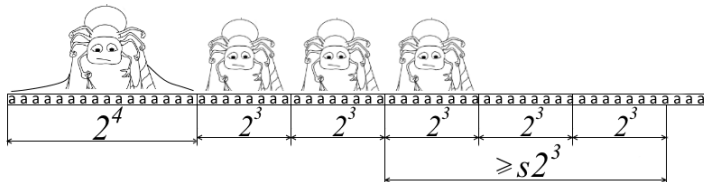


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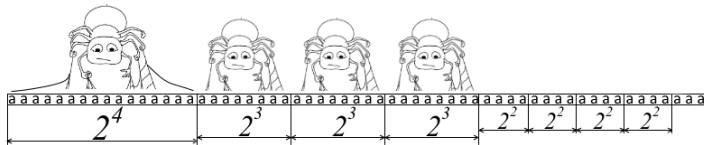


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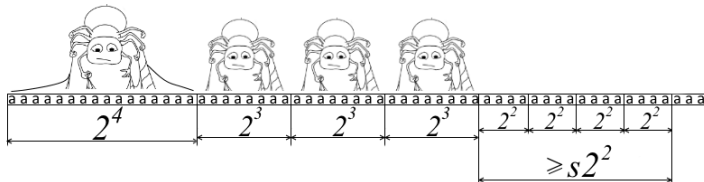


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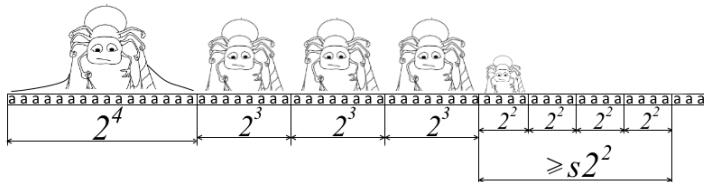


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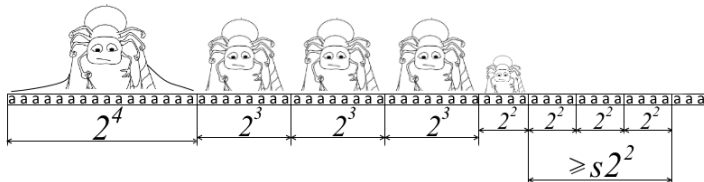


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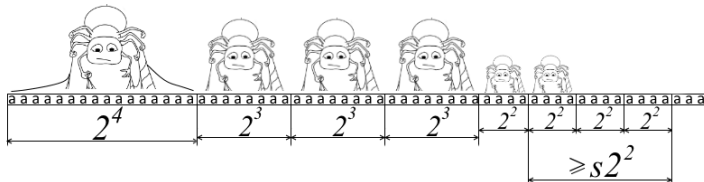


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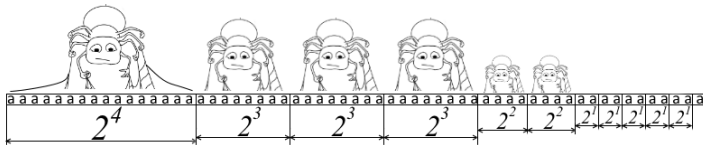


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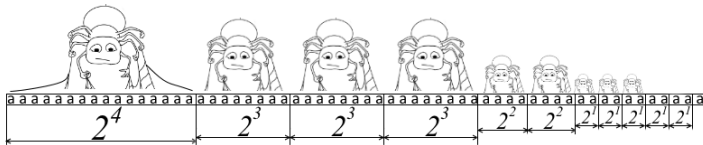


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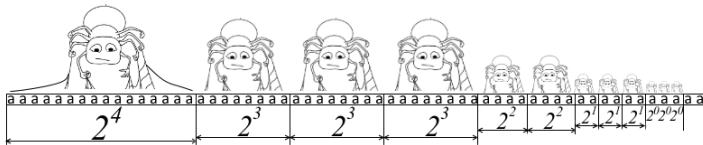


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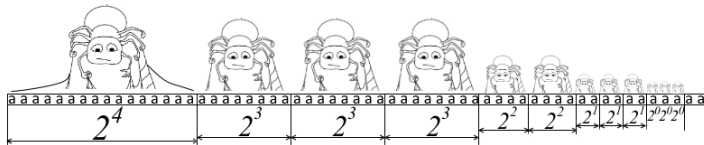


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- 1: check for e -repetitions of length $2, 3, \dots, s-1$
- 2: **for** ($k \leftarrow 0$; $s2^k \leq |t|$ **and** $|t| \bmod 2^k = 0$; $k \leftarrow k + 1$) **do**
- 3: **if** $k > 0$ **then**
- 4: remove two catchers “covering” $(|t| - s2^k .. |t| - (s-1)2^k]$
- 5: create a catcher “covering” $(|t| - s2^k .. |t| - (s-1)2^k]$

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The overall time is $O(n \log m)$ (m is the length of a longest string generated by a given sequence of n *appends* and *backtracks*)

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Lemma

Let u be the shortest unioccurent suffix of t and r be an e -repetition of t . If $t[0..|t|-2]$ is e -repetition-free, then $|u| \leq |r| < \frac{e}{e-1}|u|$.

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Three catchers can efficiently cover the range described in Lemma
(see details in the paper)

Thank you for your attention!