Searching of gapped repeats and subrepetitions in a word

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$w = a_1 \dots a_n, \quad |w| = n$ — the length of w

Def: p is a *period* of w if $a_1 \dots a_{n-p} = a_{p+1} \dots a_n$

$p(w)$ — the minimal period of w

$e(w) = \frac{|w|}{p(w)}$ — the *exponent* of w

Ex: $w = aabaa$

3, 4 and 5 — periods of w

3 — minimal period of w

$\frac{5}{3}$ — exponent of w

w — *repetition* if $e(w) \geq 2$

uu — *square*

Ex: $abcabc = (abc)^2$ — square of abc

uuu — *cube*

Ex: $abcabcabc = (abc)^3$ — cube of abc

$\underbrace{uu\ldots u}_{n}$ — *n-th power* of u, $n \geq 2$

word is *primitive* if it is not a power of some word

square uu is *primitive* if u is primitive

M.Crochemore 1981: a word of length n contains $O(n \log n)$ primitive squares, and all primitive squares in the word can be found in $O(n \log n)$ time

D.Gusfield, J.Stoye 2004: all primitive squares in a word of length n can be found in $O(n + S)$ time where S is the number of the squares (for the constant alphabet size)

(fractional) repetition:

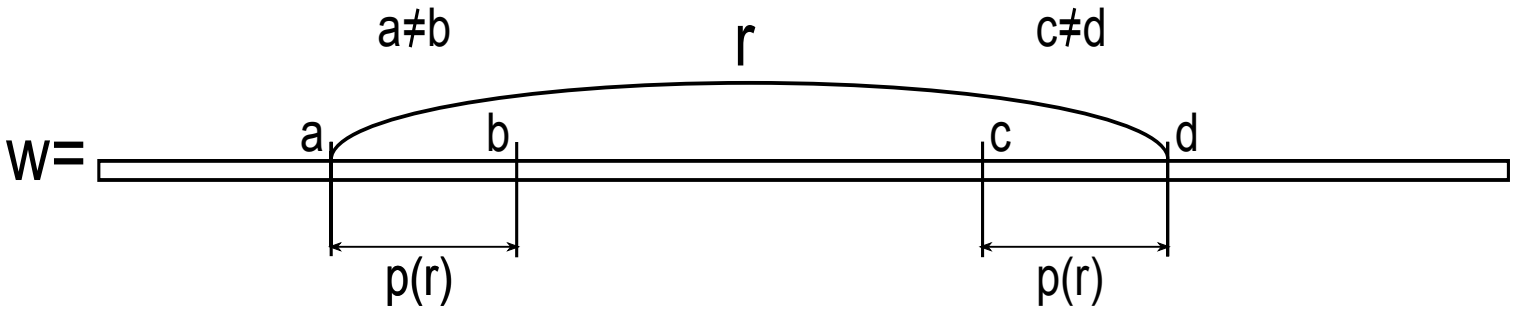
$r = \underbrace{uu \ldots u}_{k} u' = u^k u'$, where $|u| = p(r)$, $k \geq 2$, u' — prefix of u

u — *minimal root* of r

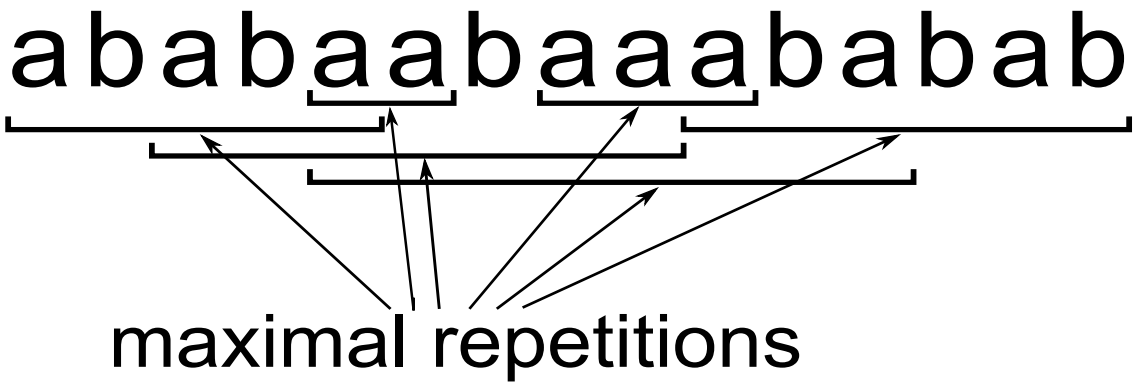
Ex: $r = abcabcabcabcab = (abc)^4 ab$

abc — *minimal* root of r

a repetition r in a word w is maximal (run) if



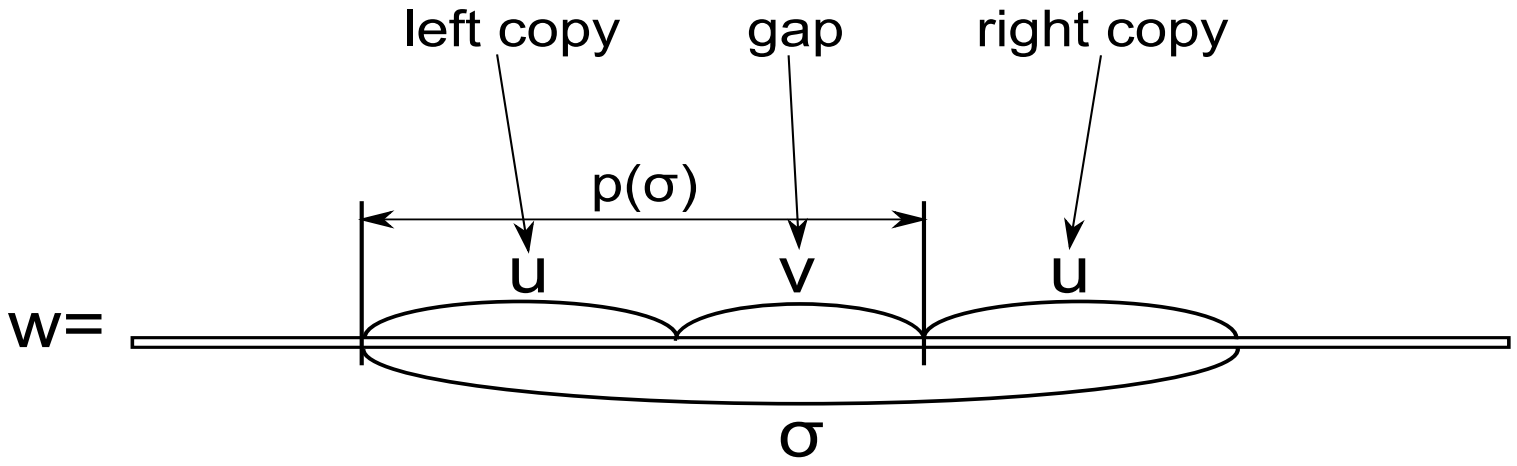
Ex:



R.Kolpakov, G.Kucherov 1999: a word of length n contains $O(n)$ maximal repetitions, and all maximal repetitions in the word can be found in $O(n)$ time (for the constant alphabet size)

M.Crochemore, L.Ilie, L.Tinta 2008: a word of length n contains no more than $1.029n$ maximal repetitions

$\sigma = uvu$ — a gapped repeat in w:

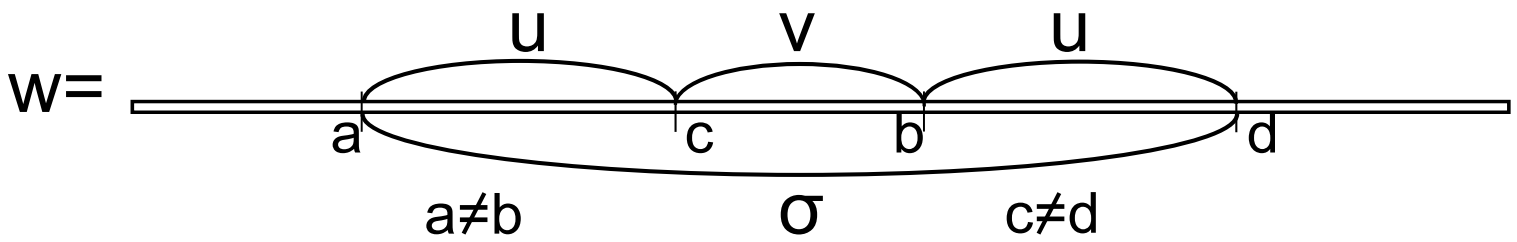


$p(\sigma)$ — the period of σ

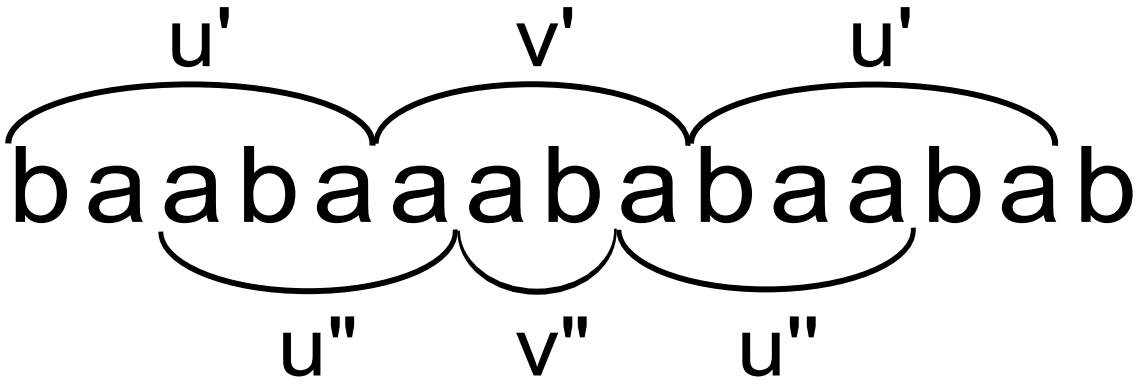
$c(\sigma) = |u|$ — the length of copies of σ

σ is α-gapped repeat if $p(\sigma) \leq \alpha c(\sigma)$

σ is maximal gapped repeat in w if



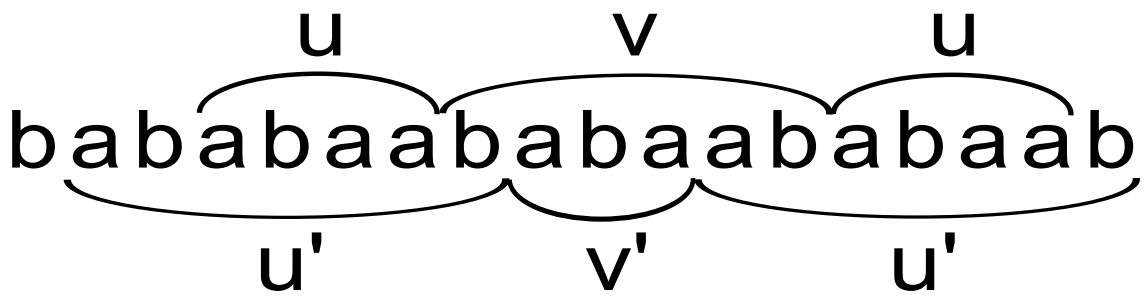
Ex:



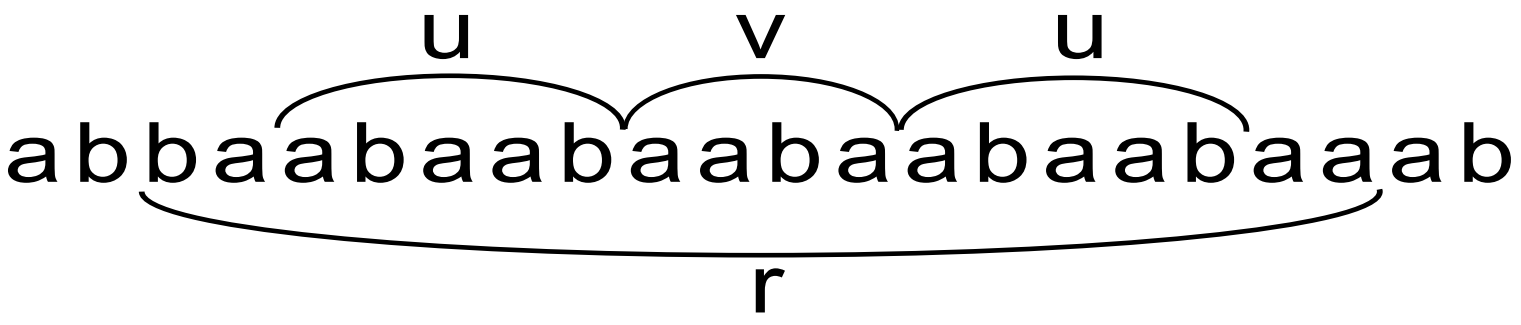
G.Brodal, R.Lyngso, C.Pedersen, J.Stoye 2000: in a word of length n all maximal gapped repeats with gap length from a given interval can be found in $O(n \log n + S)$ time where S is the output size

R.Kolpakov, G.Kucherov 2000: in a word of length n all gapped repeats with a given gap length d can be found in $O(n \log d + S)$ time where S is the output size (for the constant alphabet size)

any α-gapped repeat $\sigma = uvu$ is contained in either (uniquely defined) maximal α-gapped repeat $\sigma' = u'v'u'$ with the same period, **e.g:**

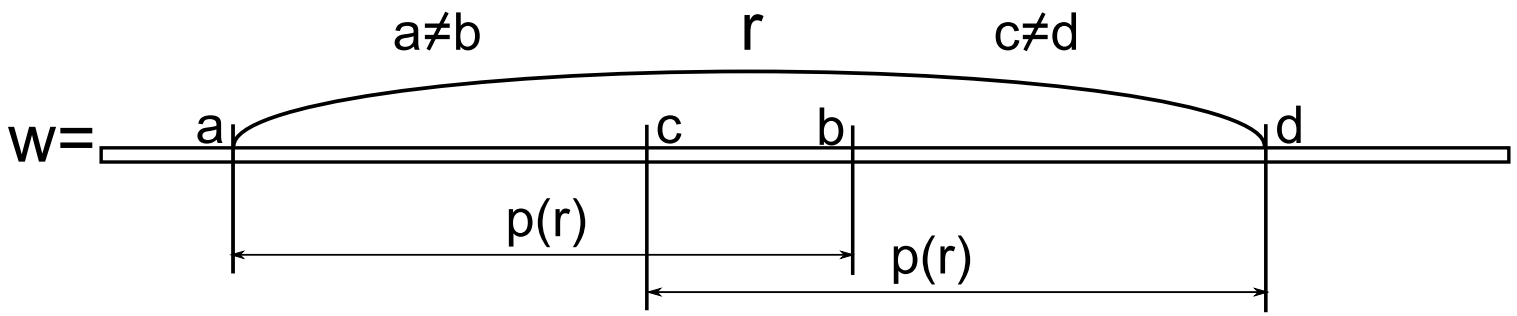


or (uniquely defined) maximal repetiton r such that $p(r)$ is a divisor of $p(\sigma)$, **e.g:**



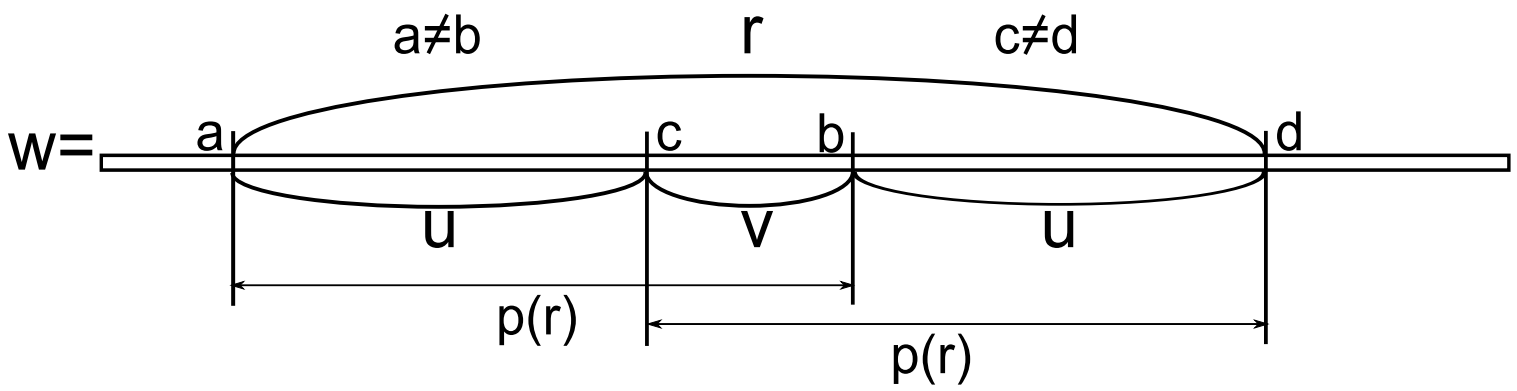
r is δ-subrepetition if $1 + \delta \leq e(r) < 2$

a subrepetition r in a word w is maximal if



R.Kolpakov, G.Kucherov, P.Ochem 2010: a word of length n contains $O(\frac{n}{\delta} \log n)$ maximal δ-subrepetitions

r — maximal δ-subrepetition in a word w



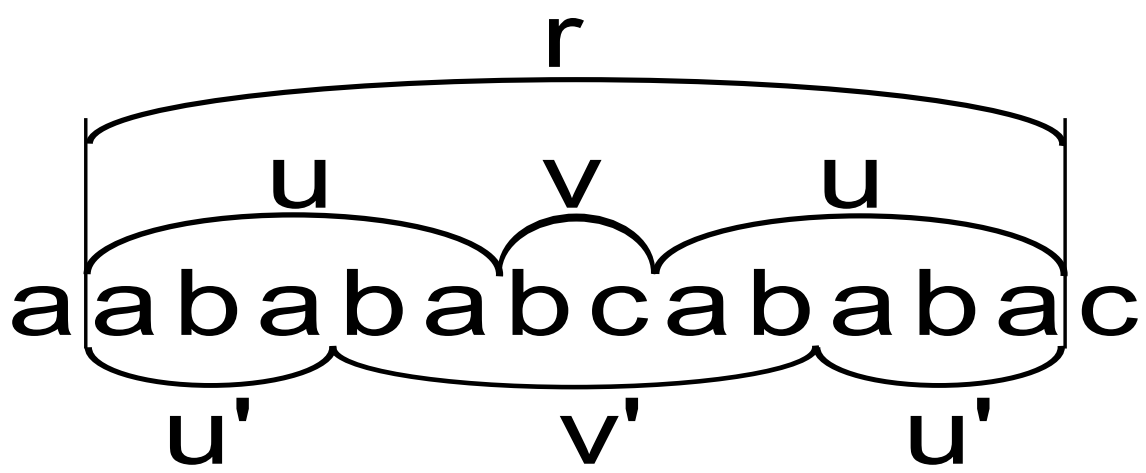
$\sigma = uvu$ — maximal $\frac{1}{\delta}$-gapped repeat *respective to* r

r and σ are the same factor in w

$\Downarrow$

r and σ are uniquely defined by each other

Ex:



$\sigma = uvu$ — maximal gapped repeat respective to r

$\sigma' = u'v'u'$ — maximal gapped repeat, s.t. r and σ' are same factor but σ' is not respective to r

thus, σ is *principal*, and σ' is not principal

number of maximal δ-subrepetitions = number of principal maximal $\frac{1}{\delta}$-gapped repeats

$$\Downarrow$$

number of maximal δ-subrepetitions $\leq$ number of maximal $\frac{1}{\delta}$-gapped repeats

Lemma: A word of length n contains $O(\alpha^2 n)$ maximal α-gapped repeats.

Corollary: A word of length n contains $O(n/\delta^2)$ maximal δ-subrepetitions.

Theorem 1: In a word of length n all maximal α-gapped repeats can be computed in $O(\alpha^2 n)$ time for the constant alphabet size and in $O(n \log n + \alpha^2 n)$ time for the general case.

computing all maximal δ-subrepetitions $\equiv$ selecting all principal repeats from all maximal $\frac{1}{\delta}$-gapped repeats

Proposition 1: A maximal α-gapped repeat σ is principal iff σ is not contained in a maximal α-gapped repeat σ' s.t. $p(\sigma') < p(\sigma)$ or in a maximal repetition r s.t. $p(r) < p(\sigma)$.

Theorem 2: In a word of length n all maximal δ-subrepetitions can be computed in $O(\frac{n \log\log n}{\delta^2})$ time for the constant alphabet size and $O(n \log n + \frac{n \log\log n}{\delta^2})$ time for the general case.

Proposition 2: A maximal gapped repeat $\sigma = uvu$ is principal iff $p(\sigma)$ is the minimal period of uvu.

T.Kociumaka, J.Radoszewski, W.Rytter, T.Walen 2012: a hash table data structure for a given word of length n can be constructed in $O(n \log n)$ expected time and allows to compute the minimal period p of a required factor u of the word in $O(\log(1 + \frac{|u|}{|u|-p}))$ time

Theorem 3: In a word of length n all maximal δ-subrepetitions can be computed in $O(n \log n + \frac{n}{\delta^2} \log \frac{1}{\delta})$ expected time.

Further research:

- improving the bounds for the numbers of maximal α-gapped repeats and maximal δ-subrepetitions in words of fixed length;
- optimal algorithms for computing all maximal α-gapped repeats and all maximal δ-subrepetitions in a word.