A *really* simple approximation of smallest grammar

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Compression and grammars

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- Increasingly popular

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Grammars based compression

- CFG defining unique word
- Straight Line Programs (SLP)

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Grammars based compression

- CFG defining unique word
- Straight Line Programs (SLP)
- easy to work on
- related to block compression

Smallest grammar

Problem

Given w return smallest CFG G_w such that $L(G_w) = w$.

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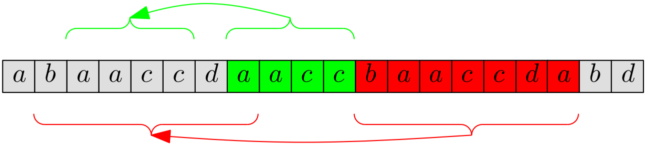
This talk

A really simple linear algorithm with this bound.

LZ77

- Represent w as $w = f_1 f_2 f_3 \dots f_\ell$.
- Each $f = w[i\,..\,i + |f|)$ is
  - **free letter** a letter *or*
  - **factor** equal to $w[j\,..\,j + |f|)$ for some $j < i$.
- size of factorisation $f_1 f_2 f_3 \dots f_\ell$ is ℓ



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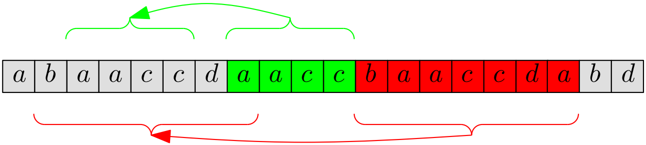
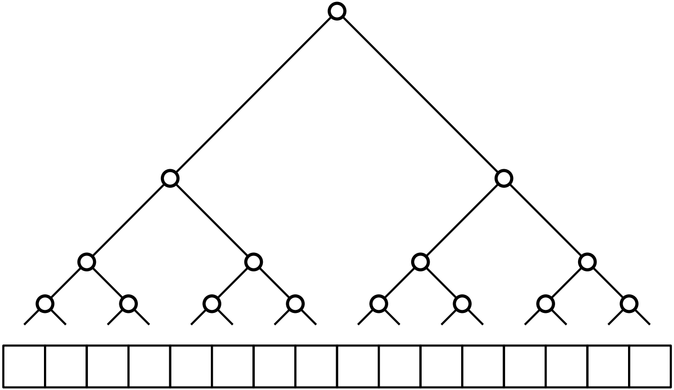


- smallest can be found in $\mathcal{O}(|w|)$
- smaller than smallest grammar

Grammar = iterated pairing



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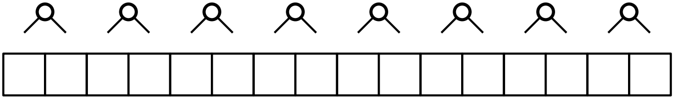

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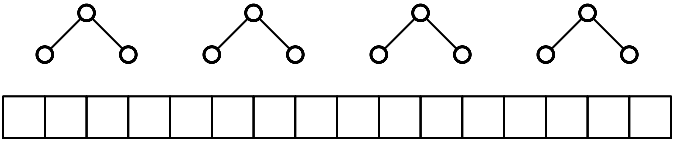

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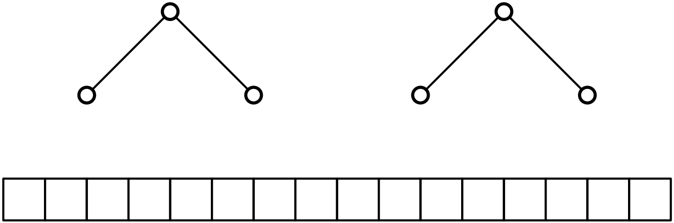

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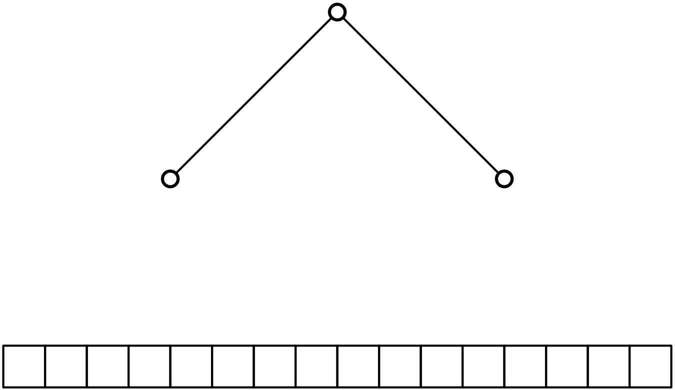

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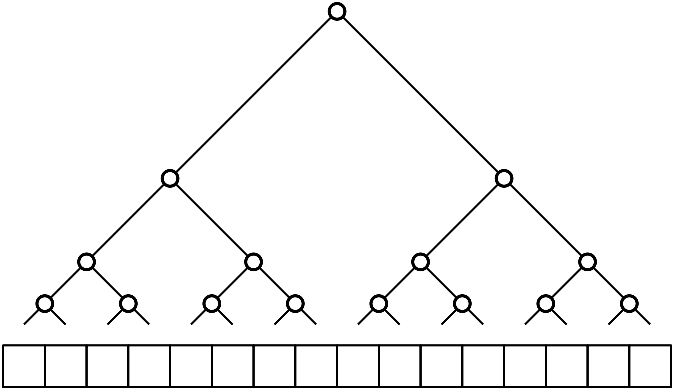
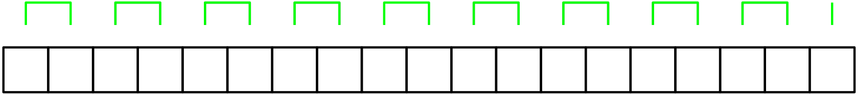


- grammar = iterated pairing
- size = different pairs (in total)

Pairing



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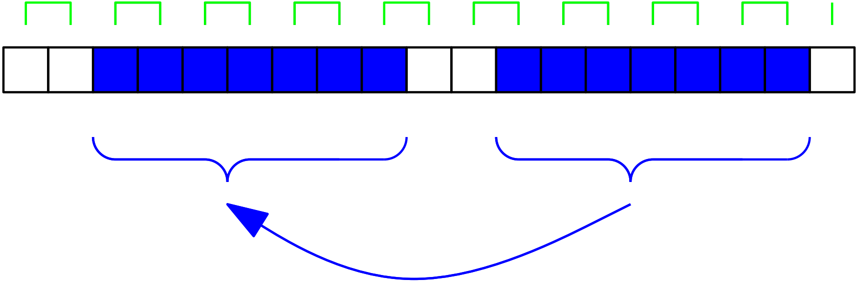

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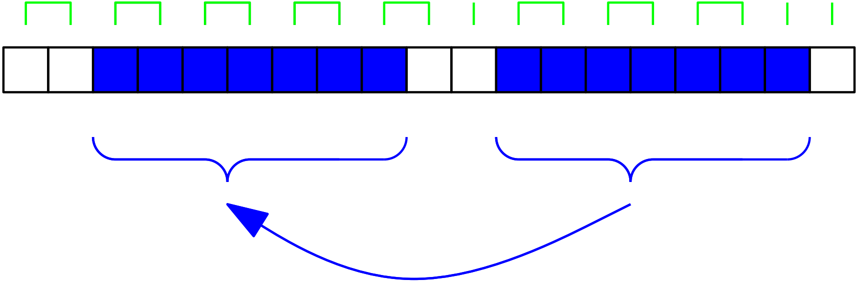

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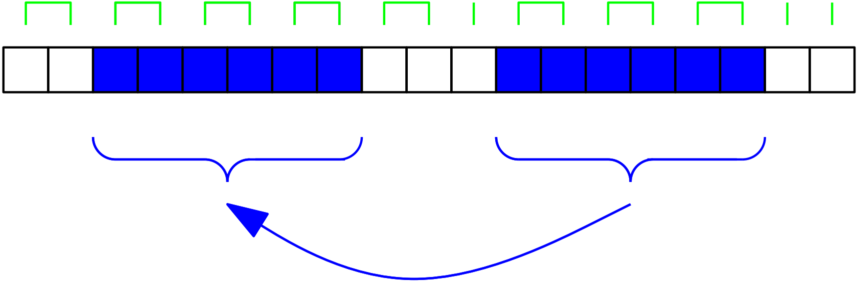

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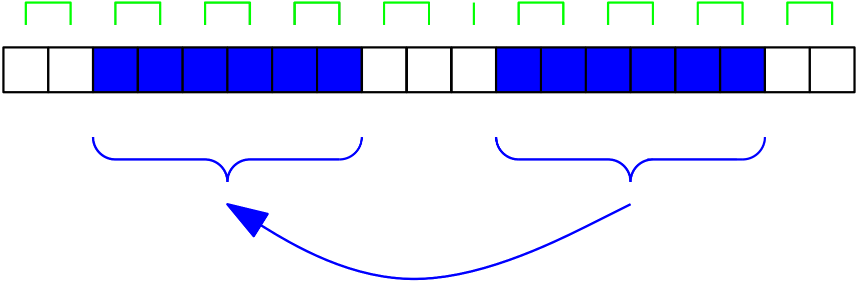

Properties

- no two consecutive letters are unpaired
- factor is paired as its definition
- first two (last two) letters of a factor are paired

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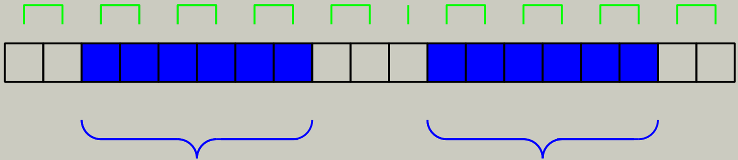
After replacing: factorisation with the same number of factors.



Pairing

while $i \leq |t|$ **do**

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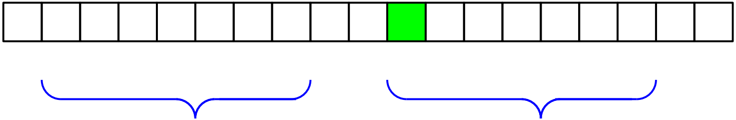

while $i \leq |t|$ **do**

    if i is the first letter of a factor **then**

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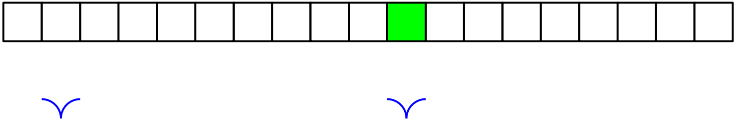

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while $i \leq |t|$ **do**
    if i is the first letter of a factor **then**
        if factor has one letter **then**
            replace it with a free letter

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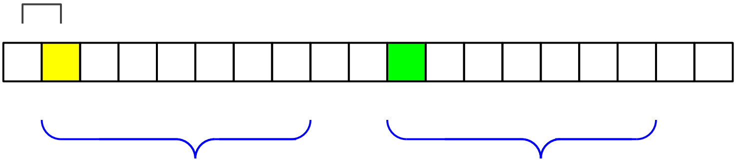

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while i ≤ |t| do
    if i is the first letter of a factor then
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        else if j is not a first letter in a pair then
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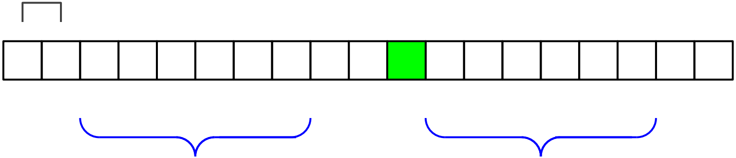

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while i ≤ |t| do
    if i is the first letter of a factor then
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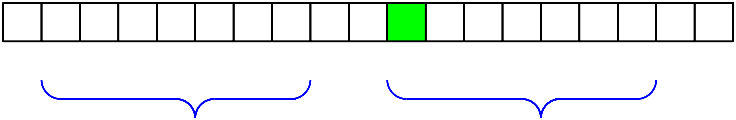

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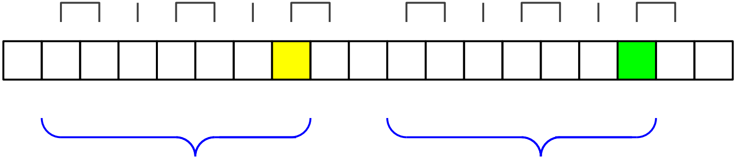

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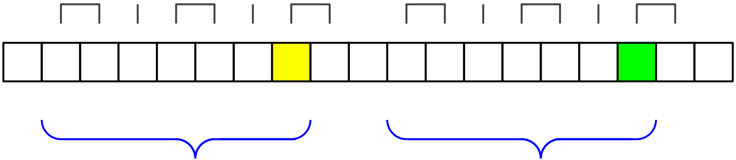

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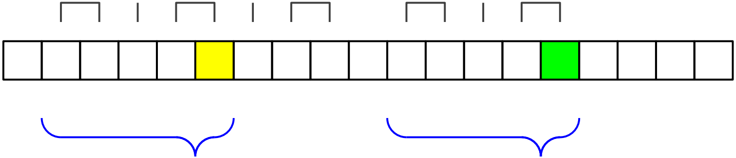

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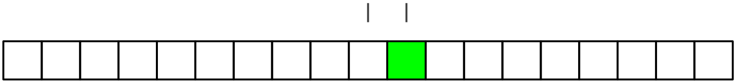
while $i \leq |t|$ **do**

    ...

    if i is a free letter **then**



```
while i ≤ |t| do
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    if i is a free letter then
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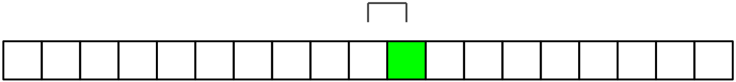

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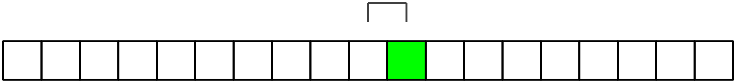
            pair them ▷ New pair!



while $i \leq |t|$ **do**

    . . .

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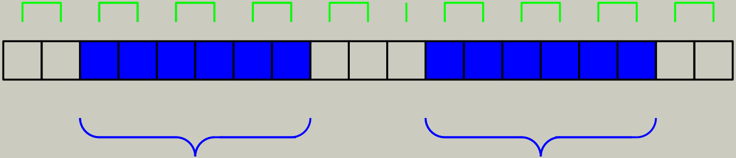
- Invariants are preserved

Pair replacement

- replace pairs with new letters
- leave unpaired letters as they are



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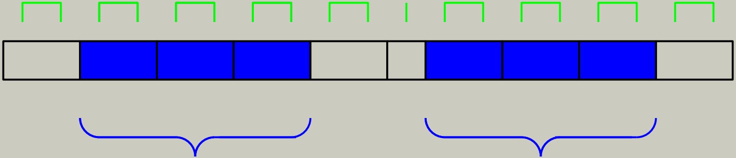

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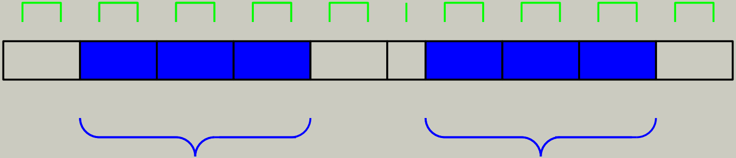

- we ‘inherit’ the factorisation
  - factors well-defined: factor and definition paired the same
    identify old and new
  - free letters: old ones or pairs of old ones

Analysis

Recall: number of nonterminals = number of different pairs

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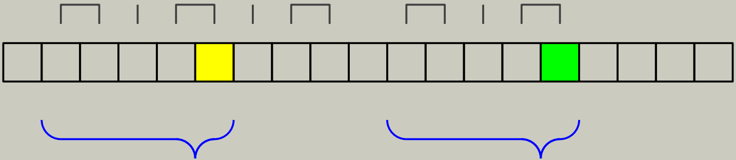

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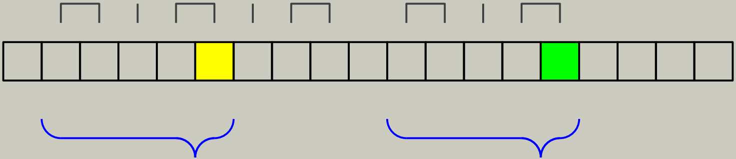
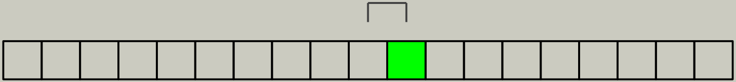


- a pair of free letters creates one new



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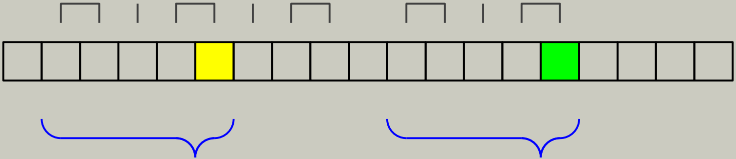

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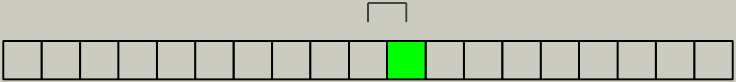

- compressed into a single free letter: decrease count by 1
- need to count number of introduced free letters

Analysis continued

Fix a factor and phase.

How many free letters?

- when replaced with a letter: one in total
- two from the left (pop until a first in a pair is found: at most 2)
- two from the right (symmetric)

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Factor f is present in $\mathcal{O}(\log |f|)$ rounds

$$\sum_{i=1}^{\ell} \log |f_i| \text{ when } \sum_{i=1}^{\ell} |f_i| = n$$

$$\mathcal{O}(\ell \log(n/\ell)) \leq \mathcal{O}(g \log(n/g))$$

Open problems

- Lower bound
  - only constant lower bound for approximation ratio
  - already for (very simple) strings
- What is the approximation bound?
- Hardness?
- simplifications (subclasses)