Shortest Unique Substring Query Revisited

Atalay Mert İleri
Bilkent University, Turkey

M. Oğuzhan Külekci
Istanbul Medipol University, Turkey

Bojian Xu
Eastern Washington University, USA

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Outline

1. Preliminary Information
2. Left Bounded Shortest Unique Substrings
3. Finding Shortest Unique Substring For All Positions
4. Experimental Results
5. Conclusion
6. Acknowledgments
7. Q&A

Shortest Unique Substring (SUS)

A substring $S[i \cdots j]$ is called **Shortest Unique Substring** covering a position k, if $i \leq k \leq j$ and there is no other unique substring $S'[i' \cdots j']$ exists such that $i' \leq k \leq j'$ and $j' - i' < j - i$

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Problem 1 (find details in paper)

Given a string location k, find the leftmost SUS covering location k.

Problem 2 (find details in paper)

Given a string location k, find all the SUSes covering location k.

Problem 3 (discussed in this talk)

Find the leftmost SUS covering every string location $1, 2, \ldots, n$.

Problem 4 (find details in paper)

Find all the SUSes covering every string location $1, 2, \ldots, n$.

Why SUS queries ?

- Distinguishing results in a text search
- Finding DNA signatures of organisms
- Extracting the event context in historical events

Preliminaries: Suffix Array and Rank Array

1	mississippi
2	ississippi
3	ssissippi
4	sissippi
5	issippi
6	ssippi
7	sippi
8	ippi
9	ppi
10	pi
11	i

Preliminaries: Suffix Array and Rank Array

1	mississippi		11	i
2	ississippi		8	ippi
3	ssissippi		5	issippi
4	sissippi		2	ississippi
5	issippi		1	mississippi
6	ssippi		10	pi
7	sippi	$\Rightarrow$	9	ppi
8	ippi		7	sippi
9	ppi		4	sissippi
10	pi		6	ssippi
11	i		3	ssissippi

Preliminaries: Suffix Array and Rank Array

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$\Rightarrow$

11	i
8	ippi
5	issippi
2	ississippi
1	mississippi
10	pi
9	ppi
7	sippi
4	sissippi
6	ssippi
3	ssissippi

SA	11	8	5	2	1	10	9	7	4	6	3
RANK	5	4	11	9	3	10	8	2	7	6	1

Preliminaries: Longest Common Prefix Array

11	i
8	ippi
5	issippi
2	ississippi
1	mississippi
10	pi
9	ppi
7	sippi
4	sissippi
6	ssippi
3	ssissippi

LCP	0	1	1	4	0	0	1	0	2	1	3

Left Bounded Shortest Unique Substring

Definition

For a particular string location $k \in \{1, 2, \ldots, n\}$, the **left-bounded shortest unique substring (LSUS) starting at location k**, denoted as $\mathbf{LSUS_k}$, is a unique substring $S[k \ldots j]$, such that either $k = j$ or any proper prefix of $S[k \ldots j]$ is not unique.

Examples:

mississippi $\longrightarrow LSUS_1 = S[1, 1]$
mississippi $\longrightarrow LSUS_2 = S[2, 6]$
$\vdots$
mississippi $\longrightarrow LSUS_{10} = S[10, 11]$
mississippi $\longrightarrow LSUS_{11}$ does not exist

Fast computation of LSUS'es from LCP array

For $i = 1, 2, \ldots, n$:

$$LSUS_i = \begin{cases} S[i \ldots i + L_i], & \text{if} \quad i + L_i \leq n \\ \textit{not existing}, & \text{otherwise} \end{cases}$$

where $L_i = \max\{LCP[RANK[i]], LCP[RANK[i] + 1]\}$.

Fast computation of LSUS'es from LCP array

mississippi

RANK	5	4	11	9	3	10	8	2	7	6	1
LCP	0	1	1	4	0	0	1	0	2	1	3
L_i	0										
$LSUS_i$	m										

Fast computation of LSUS'es from LCP array

mississippi

RANK	5	4	11	9	3	10	8	2	7	6	1
LCP	0	1	1	4	0	0	1	0	2	1	3
L_i	0	4									
$LSUS_i$	m	issis									

Fast computation of LSUS'es from LCP array

mississippi

RANK	5	4	11	9	3	10	8	2	7	6	1
LCP	0	1	1	4	0	0	1	0	2	1	3
L_i	0	4	3								
$LSUS_i$	m	issis	ssis								

Fast computation of LSUS'es from LCP array

mississippi

RANK	5	4	11	9	3	10	8	2	7	6	1
LCP	0	1	1	4	0	0	1	0	2	1	3
L_i	0	4	3	2	4	3	2	1	1		
$LSUS_i$	m	issis	ssis	sis	issip	ssip	sip	ip	pp		

Fast computation of LSUS'es from LCP array

mississippi

RANK	5	4	11	9	3	10	8	2	7	6	1
LCP	0	1	1	4	0	0	1	0	2	1	3
L_i	0	4	3	2	4	3	2	1	1	1	
$LSUS_i$	m	issis	ssis	sis	issip	ssip	sip	ip	pp	pi	

Fast computation of LSUS'es from LCP array

mississippi

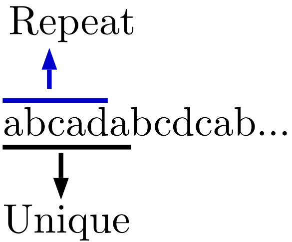
RANK	5	4	11	9	3	10	8	2	7	6	1
LCP	0	1	1	4	0	0	1	0	2	1	3
L_i	0	4	3	2	4	3	2	1	1	1	1
$LSUS_i$	m	issis	ssis	sis	issip	ssip	sip	ip	pp	pi	-

Properties of LSUS

- $LSUS_1$ always exists, because at least it can be the whole string, which is unique.

Properties of LSUS

- $LSUS_1$ always exists, because at least it can be the whole string, which is unique.
- $LSUS_i$ cannot end before $LSUS_{i-1}$



Finding leftmost SUS for all positions: Overview

$$SUS_i = \begin{cases} SUS_{i-1} + S[i], & \text{if} \quad |SUS_{i-1}| + 1 \leq |SLS_i| \\ SLS_i, & \text{if} \quad |SUS_{i-1}| + 1 > |SLS_i| \end{cases}$$

SLS_i

Leftmost shortest LSUS covering string location i.

abcabcbcab

abcabcbcab

abcabcbcab

abcabcbcab

abcabcbcab

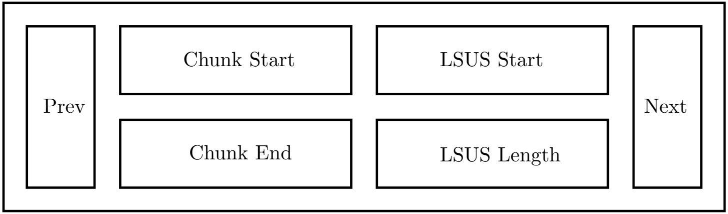
abcabcbcab

abcabcbcab

abcabcbcab

abcabcbcab

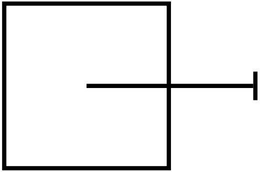
Finding the candidate: Data structure



Example Run

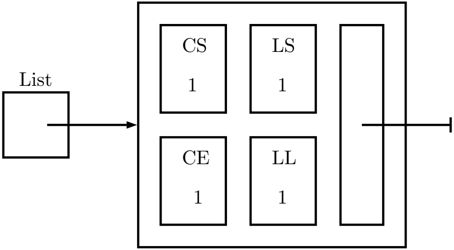
S = mississippi

List



Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

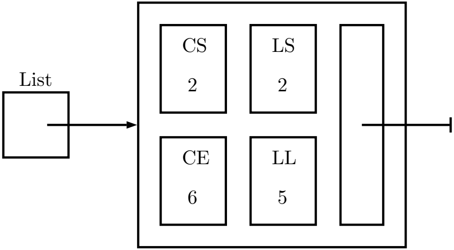


$$SUS_0 = -$$
$$SLS_1 = (1, 1)$$
$$SUS_1 = (1, 1)$$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

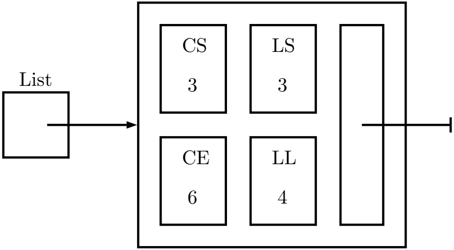


$SUS_1 = (1, 1)$
$SLS_2 = (2, 5)$
$SUS_2 = (1, 2)$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

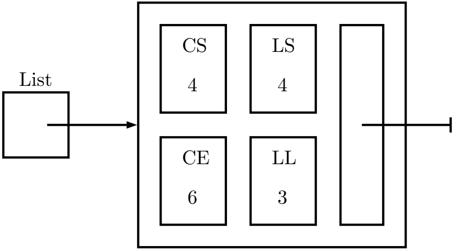


$SUS_2 = (1, 2)$
$SLS_3 = (3, 4)$
$SUS_3 = (1, 3)$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

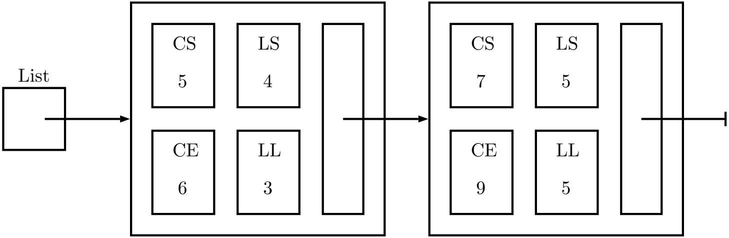


$SUS_3 = (1, 3)$
$SLS_4 = (4, 3)$
$SUS_4 = (4, 3)$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

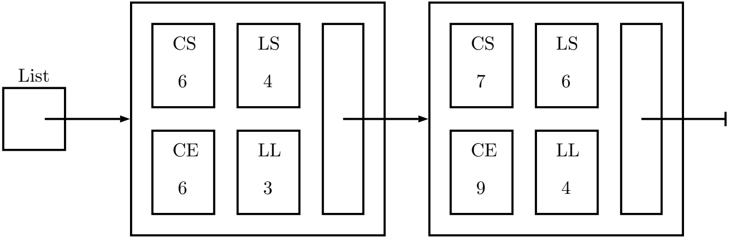


$$SUS_4 = (4, 3)$$
$$SLS_5 = (4, 3)$$
$$SUS_5 = (4, 3)$$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

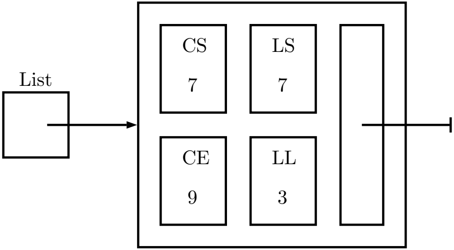


$$SUS_5 = (4, 3)$$
$$SLS_6 = (4, 3)$$
$$SUS_6 = (4, 3)$$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

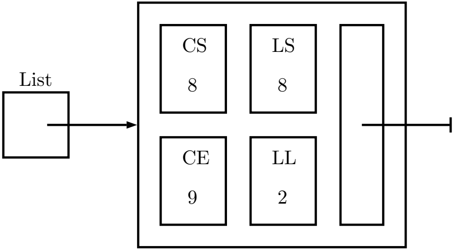


$$SUS_6 = (4, 3)$$
$$SLS_7 = (7, 3)$$
$$SUS_7 = (7, 3)$$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

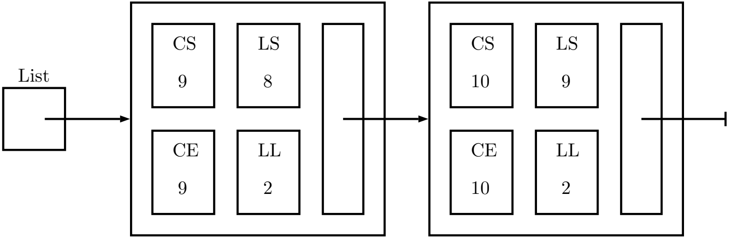


$SUS_7 = (7, 3)$
$SLS_8 = (8, 2)$
$SUS_8 = (8, 2)$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

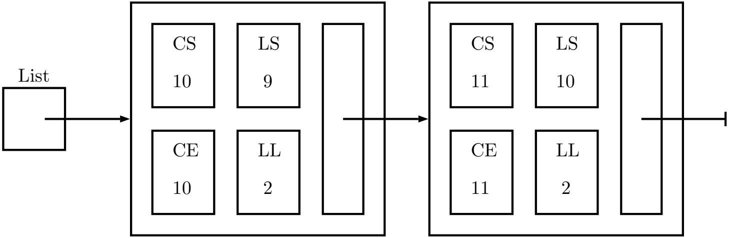


$$SUS_8 = (8,2)$$
$$SLS_9 = (8,2)$$
$$SUS_9 = (8,2)$$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i

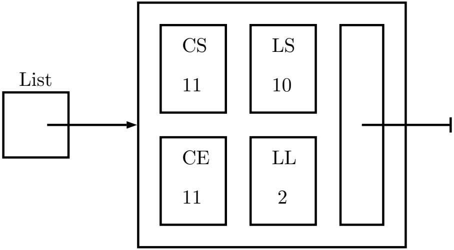


$SUS_9 = (8, 2)$
$SLS_{10} = (9, 2)$
$SUS_{10} = (9, 2)$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Example Run

1	2	3	4	5	6	7	8	9	10	11
m	i	s	s	i	s	s	i	p	p	i



$$SUS_{10} = (9, 2)$$
$$SLS_{11} = (10, 2)$$
$$SUS_{11} = (10, 2)$$

CS: Chunk Start
CE: Chunk End
LS: LSUS Start
LL: LSUS Length

Time Complexity

	Time
Derivation of LCP and Rank array	$O(n)$
Calculation of LSUS	$O(1)$
Maintenance of data structure	$O(1)$
Finding SLS	$O(1)$
Determining SUS	$O(1)$
Number of iterations:	n

Total Time: $O(n) + n \cdot (O(1) + O(1) + O(1) + O(1)) = O(n)$

Space Complexity

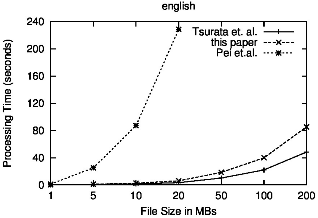
	Memory (in words)
LCP array	$O(n)$
Rank array	$O(n)$
Data structure	$O(n)$
Previous SUS	$O(1)$

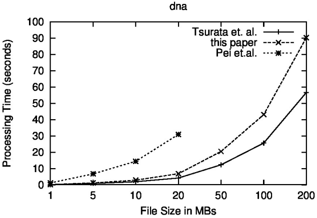
Total Memory: $O(n) + O(n) + O(n) + O(1) = O(n)$

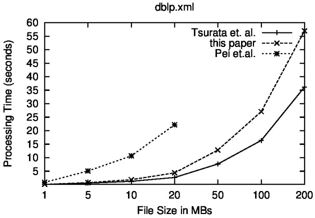
Involved work in experimental study

- J. Pei, W. C. H. Wu, M. Y. Yeh, "On shortest unique substring queries", ICDE 2013
- K. Tsuruta, S. Inenaga, H. Bannai, M. Takeda, "Shortest unique substrings queries in optimal time", SOFSEM 2014

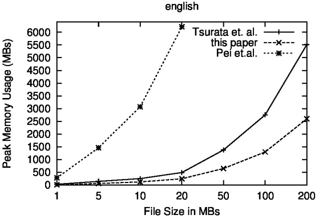
Time Measurements

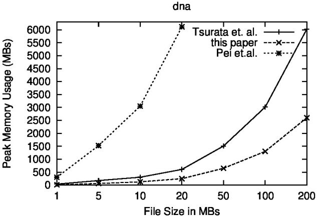


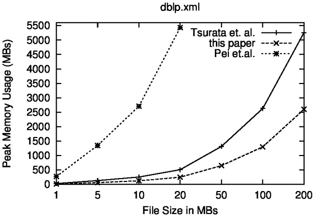




Memory Measurements







Conclusion

- We developed O(n) time and space algorithms for finding SUS covering one position and all positions.
- Our work improved the prior work of Pei et al. 8 times in terms of time and 20 times in terms of memory.

Acknowledgments

I want to thank them for their monetary support for my trip to CPM2014.

CPM Organizing Commitee

Bilim Akademisi

Thanks !

Questions ?