Linear Time Lempel-Ziv Factorization: Simple, Fast, Small

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Outline

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

$p_7 = 2$ $\ell_7 = 3$

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

Example

1 2 3 4 5 6 7 8 9 10 11 12

X = b a b b a b a b b b a b

$p_{10} = 1$ $\ell_{10} = 3$

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

$p_{10} = 4$

$\ell_{10} = 3$

Example

	1	2	3	4	5	6	7	8	9	10	11	12
X =	b	a	b	b	a	b	a	b	b	b	a	b

↑ $p_{10} = 6$

← - - - - - → $\ell_{10} = 3$

Example

1 2 3 4 5 6 7 8 9 10 11 12

X = b a b b a b a b b b a b

$p_{10} = 6$ $\ell_{10} = 3$

Definition

Pairs (p_i, ℓ_i) define the LPF array: $\mathrm{LPF}[i] = (p_i, \ell_i)$.

LPF array

i	LPF p_i	LPF ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1 b	2 a	3 b	4 b	5 a	6 b	7 a	8 b	9 b	10 b	11 a	12 b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

Lempel-Ziv Factorization

	LPF		1	2	3	4	5	6	7	8	9	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

Lempel-Ziv Factorization

	LPF		1	2	3	4	5	6	7	8	9	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0)

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0)

Lempel-Ziv Factorization

	LPF		1	2	3	4	5	6	7	8	9	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0)

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1 b	2 a	3 b	4 b	5 a	6 b	7 a	8 b	9 b	10 b	11 a	12 b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	**1**	**1**	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0)

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	**1**	**1**	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1)

Lempel-Ziv Factorization

i	p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	**1**	**3**	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1)

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	**1**	**3**	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3)

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	**2**	**3**	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3)

Lempel-Ziv Factorization

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	**2**	**3**	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

Lempel-Ziv Factorization

i	p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	**4**	**3**	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LPF

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

Lempel-Ziv Factorization

i	LPF p_i	LPF ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			(b)	(a)	(b)	(b	a	b)	(a	b	b)	(b	a	b)
1	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	**4**	**3**	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3),(4,3)

Lempel-Ziv Factorization

	LPF		1	2	3	4	5	6	7	8	9	10	11	12
i	p_i	ℓ_i	(b)	(a)	(b)	(b	a	b)	(a	b	b)	(b	a	b)
1	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3),(4,3)

Outline

1. Introduction
2. Existing solutions
3. 2n log n algorithm

Existing solutions

- Space excludes the input and output (both of size $n \log \sigma$).

Algorithm	Extra space
Abouelhoda et al., 2004	$4n \log n$
Chen et al. (CPS1), 2007	$3n \log n$
Crochemore and Ilie, 2008	$(3n + \sqrt{n}) \log n$
Ohlebusch and Gog, 2011	$3n \log n$
Goto and Bannai (BGL), 2013	$3n \log n$

Our contribution

Two new linear time algorithms:

Our contribution

Two new linear time algorithms:

1. K3: $3n \log n$ bits of extra space

Our contribution

Two new linear time algorithms:

1. K3: $3n \log n$ bits of extra space
   - minimizes the number of cache misses

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   - minimizes the number of cache misses
   - fastest, when the input is not highly repetitive

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Two new linear time algorithms:

1. K3: $3n \log n$ bits of extra space
   - minimizes the number of cache misses
   - fastest, when the input is not highly repetitive

Algorithm K3

$\mathsf{SA}[0] \leftarrow \mathsf{SA}[n+1] \leftarrow \mathit{top} \leftarrow 0$
for $i \leftarrow 1$ **to** $n+1$ **do**
  while $\mathsf{SA}[\mathit{top}] > \mathsf{SA}[i]$ **do**
    $\mathsf{NSV}[\mathsf{SA}[\mathit{top}]] \leftarrow \mathsf{SA}[i]$
    $\mathsf{PSV}[\mathsf{SA}[\mathit{top}]] \leftarrow \mathsf{SA}[\mathit{top}-1]$
    $\mathit{top} \leftarrow \mathit{top} - 1$
  $\mathit{top} \leftarrow \mathit{top} + 1$
  $\mathsf{SA}[\mathit{top}] \leftarrow \mathsf{SA}[i]$
$i \leftarrow 1$
while $i \leq n$ **do**
  $i \leftarrow \texttt{LZ-Factor}(i, \mathsf{NSV}[i], \mathsf{PSV}[i])$

Procedure $\texttt{LZ-Factor}(i, nsv, psv)$

$\ell_{nsv} \leftarrow \mathsf{lcp}(i, nsv)$
$\ell_{psv} \leftarrow \mathsf{lcp}(i, psv)$
if $\ell_{nsv} > \ell_{psv}$ **then**
  $(p, \ell) \leftarrow (nsv,\ \ell_{nsv})$
else
  $(p, \ell) \leftarrow (psv,\ \ell_{psv})$
if $\ell = 0$ **then** $p \leftarrow \mathsf{X}[i]$
output factor (p, ℓ)
return $i + \max(\ell, 1)$

Our contribution

Two new linear time algorithms:

1. K3: $3n \log n$ bits of extra space
   - minimizes the number of cache misses
   - fastest, when the input is not highly repetitive
2. K2: $2n \log n$ bits of extra space

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   - most space efficient linear algorithm for LZ77

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   - minimizes the number of cache misses
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2. K2: $2n \log n$ bits of extra space
   - most space efficient linear algorithm for LZ77
   - based on combinatorics of suffix arrays

Outline

2n log n algorithm

i	LPF p_i	LPF ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
2	⊥	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

	LPF		1	2	3	4	5	6	7	8	9	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	0	b	a	b	b	a	b	a	b	b	b	a	b
3	1	1	b	a	b	b	a	b	a	b	b	b	a	b
4	1	3	b	a	b	b	a	b	a	b	b	b	a	b
5	2	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	4	b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3	2	b	a	b	b	a	b	a	b	b	b	a	b
9	3	4	b	a	b	b	a	b	a	b	b	b	a	b
10	4	3	b	a	b	b	a	b	a	b	b	b	a	b
11	5	2	b	a	b	b	a	b	a	b	b	b	a	b
12	6	1	b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥		b	a	b	b	a	b	a	b	b	b	a	b
2	⊥		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2		b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥		b	a	b	b	a	b	a	b	b	b	a	b
2	⊥		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2		b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

	LPF		1	2	3	4	5	6	$	8	9	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$		b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2		b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

i	LPF p_i	LPF ℓ_i	1 2 3 4 5 6 \$ \$ 9 10 11 12 b a b b a b a b b b a b
1	$\perp$		b a b b a b a b b b a b
2	$\perp$		b a b b a b a b b b a b
3	1		b a b b a b a b b b a b
4	1		b a b b a b a b b b a b
5	2		b a b b a b a b b b a b
6	1		b a b b a b a b b b a b
7	2		b a b b a b a b b b a b
8	3		b a b b a b a b b b a b
9	3		b a b b a b a b b b a b
10	4		b a b b a b a b b b a b
11	5		b a b b a b a b b b a b
12	6		b a b b a b a b b b a b

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

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The p_i component of LPF array is enough to compute LZ77 parsing

i	LPF p_i	LPF ℓ_i	1 2 3 4 5 6 $ $ $ 10 11 12 b a b b a b a b b b a b
1	⊥		b a b b a b a b b b a b
2	⊥		b a b b a b a b b b a b
3	1		b a b b a b a b b b a b
4	1		b a b b a b a b b b a b
5	2		b a b b a b a b b b a b
6	1		b a b b a b a b b b a b
7	2		b a b b a b a b b b a b
8	3		b a b b a b a b b b a b
9	3		b a b b a b a b b b a b
10	4		b a b b a b a b b b a b
11	5		b a b b a b a b b b a b
12	6		b a b b a b a b b b a b

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

i	LPF p_i	LPF ℓ_i	1	2	3	4	5	6	$	$	$	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥		b	a	b	b	a	b	a	b	b	b	a	b
2	⊥		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	≠	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing

	LPF		1	2	3	4	5	6	$	$	$	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$		b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing in linear time.

	LPF		1	2	3	4	5	6	$	$	$	10	11	12
i	p_i	ℓ_i	b	a	b	b	a	b	a	b	b	b	a	b
1	⊥		b	a	b	b	a	b	a	b	b	b	a	b
2	⊥		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

2n log n algorithm

Observation

The p_i component of LPF array is enough to compute LZ77 parsing in linear time.

Goal

Space efficient computation of all p_i values.

i	LPF p_i	LPF ℓ_i	1 b	2 a	3 b	4 b	5 a	6 b	$ a	$ b	$ b	10 b	11 a	12 b
1	$\perp$		b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2	3	b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

2n log n algorithm

Goal

Space efficient computation of all p_i values.

2n log n algorithm

Goal

Space efficient computation of all p_i values.

1. Consider text position i, e.g., let $i = 4$

SA	
10	a a b
11	a b
5	a b a b b a a b
7	a b b a a b
2	a b b a b a b b a a b
12	b
9	b a a b
4	b a b a b b a a b
6	b a b b a a b
1	b a b b a b a b b a a b
8	b b a a b
3	b b a b a b b a a b

2n log n algorithm

Goal

Space efficient computation of all p_i values.

1. Consider text position i, e.g., let $i = 4$
2. Locate in SA closest smaller elements

SA	
10	a a b
11	a b
5	a b a b b a a b
7	a b b a a b
2	a b b a b a b b a a b
12	b
9	b a a b
4	b a b a b b a a b
6	b a b b a a b
1	b a b b a b a b b a a b
8	b b a a b
3	b b a b a b b a a b

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1. Consider text position i, e.g., let $i = 4$
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SA	
10	a a b
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12	b
9	b a a b
4	b a b a b b a a b
6	b a b b a a b
1	b a b b a b a b b a a b
8	b b a a b
3	b b a b a b b a a b

2n log n algorithm

Goal

Space efficient computation of all p_i values.

1. Consider text position i, e.g., let $i = 4$
2. Locate in SA closest smaller elements

	SA	
	10	a a b
	11	a b
	5	a b a b b a a b
	7	a b b a a b
$\mathsf{PSV}[4] =$	2	a b b a b a b b a a b
	12	b
	9	b a a b
	4	b a b a b b a a b
	6	b a b b a a b
$\mathsf{NSV}[4] =$	1	b a b b a b a b b a a b
	8	b b a a b
	3	b b a b a b b a a b

2n log n algorithm

Goal

Space efficient computation of all p_i values.

1. Consider text position i, e.g., let $i = 4$
2. Locate in SA closest smaller elements

Lemma [Crochemore, Ilie]

Either PSV$[i]$ or NSV$[i]$ is a valid choice for p_i

	SA	
	10	a a b
	11	a b
	5	a b a b b a a b
	7	a b b a a b
PSV$[4]$ =	2	a b b a b a b b a a b
	12	b
	9	b a a b
	4	b a b a b b a a b
	6	b a b b a a b
NSV$[4]$ =	1	b a b b a b a b b a a b
	8	b b a a b
	3	b b a b a b b a a b

2n log n algorithm

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Space efficient computation of all p_i values.

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Either PSV$[i]$ or NSV$[i]$ is a valid choice for p_i

Not quite what we wanted...

	SA	
	10	a a b
	11	a b
	5	a b a b b a a b
	7	a b b a a b
PSV$[4]$ =	2	a b b a b a b b a a b
	12	b
	9	b a a b
	4	b a b a b b a a b
	6	b a b b a a b
NSV$[4]$ =	1	b a b b a b a b b a a b
	8	b b a a b
	3	b b a b a b b a a b

2n log n algorithm

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Space efficient computation of all p_i values.

1. Consider text position i, e.g., let $i = 4$
2. Locate in SA closest smaller elements

Lemma [Crochemore, Ilie]

Either PSV$[i]$ or NSV$[i]$ is a valid choice for p_i

Not quite what we wanted...

...but sufficient for LZ77.

	SA	
	10	a a b
	11	a b
	5	a b a b b a a b
	7	a b b a a b
PSV$[4]$ =	2	a b b a b a b b a a b
	12	b
	9	b a a b
	4	b a b a b b a a b
	6	b a b b a a b
NSV$[4]$ =	1	b a b b a b a b b a a b
	8	b b a a b
	3	b b a b a b b a a b

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

1. Consider text position i, e.g., let $i = 4$
2. Locate in SA closest smaller elements

Lemma [Crochemore, Ilie]

Either PSV$[i]$ or NSV$[i]$ is a valid choice for p_i

Not quite what we wanted...

...but sufficient for LZ77.

	SA	
	10	a a b
	11	a b
	5	a b a b b a a b
	7	a b b a a b
PSV$[4]$ =	2	a b b a b a b b a a b
	12	b
	9	b a a b
	4	b a b a b b a a b
	6	b a b b a a b
NSV$[4]$ =	1	b a b b a b a b b a a b
	8	b b a a b
	3	b b a b a b b a a b

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

i	LPF p_i	ℓ_i	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	⊥		b	a	b	b	a	b	a	b	b	b	a	b
2	⊥		b	a	b	b	a	b	a	b	b	b	a	b
3	1		b	a	b	b	a	b	a	b	b	b	a	b
4	1		b	a	b	b	a	b	a	b	b	b	a	b
5	2		b	a	b	b	a	b	a	b	b	b	a	b
6	1		b	a	b	b	a	b	a	b	b	b	a	b
7	2		b	a	b	b	a	b	a	b	b	b	a	b
8	3		b	a	b	b	a	b	a	b	b	b	a	b
9	3		b	a	b	b	a	b	a	b	b	b	a	b
10	4		b	a	b	b	a	b	a	b	b	b	a	b
11	5		b	a	b	b	a	b	a	b	b	b	a	b
12	6		b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

i	PSV	NSV	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1	$\perp$	$\perp$	b	a	b	b	a	b	a	b	b	b	a	b
2	$\perp$	1	b	a	b	b	a	b	a	b	b	b	a	b
3	1	$\perp$	b	a	b	b	a	b	a	b	b	b	a	b
4	2	1	b	a	b	b	a	b	a	b	b	b	a	b
5	$\perp$	2	b	a	b	b	a	b	a	b	b	b	a	b
6	1	3	b	a	b	b	a	b	a	b	b	b	a	b
7	2	4	b	a	b	b	a	b	a	b	b	b	a	b
8	3	$\perp$	b	a	b	b	a	b	a	b	b	b	a	b
9	4	3	b	a	b	b	a	b	a	b	b	b	a	b
10	7	4	b	a	b	b	a	b	a	b	b	b	a	b
11	10	5	b	a	b	b	a	b	a	b	b	b	a	b
12	7	4	b	a	b	b	a	b	a	b	b	b	a	b

LZ77:

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

i	PSV	NSV	1 b	2 a	3 b	4 b	5 a	6 b	7 a	8 b	9 b	10 b	11 a	12 b
1														
2														
3														
4														
5														
6														
7														
8														
9														
10														
11														
12														

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

i	PSV	NSV	1	2	3	4	5	6	7	8	9	10	11	12
			b	a	b	b	a	b	a	b	b	b	a	b
1														
2														
3														
4														
5														
6														
7	2	5	b	a	b	b	a	b	a	b	b	b	a	b
8														
9														
10														
11														
12														

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2n log n algorithm

Goal

Space efficient simulation of all p_i values.

i	PSV	NSV	1 2 3 4 5 6 7 8 9 10 11 12 b a b b a b a b b b a b
1			
2			
3			
4			
5			
6			
7	2	5	b a b b a b a b b b a b
8			
9			
10			
11			
12			

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			b	a	b	b	a	b	a	b	b	b	a	b
1														
2														
3														
4														
5														
6														
7	2	5	b	a	b	b	a	b	a	b	b	b	a	b
8														
9														
10														
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2n log n algorithm

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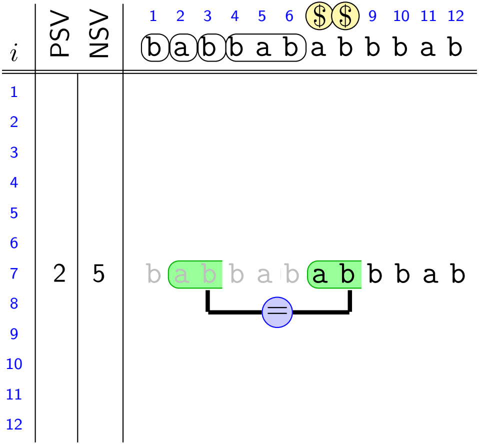
Space efficient simulation of all p_i values.



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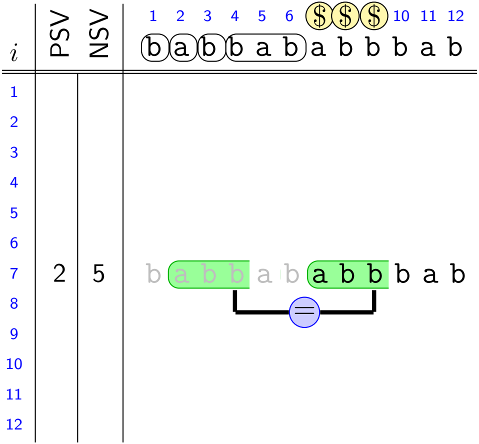

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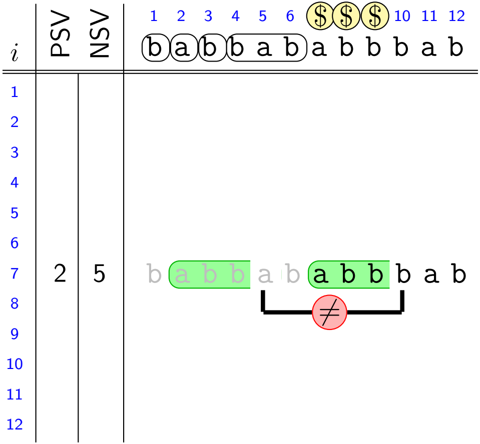

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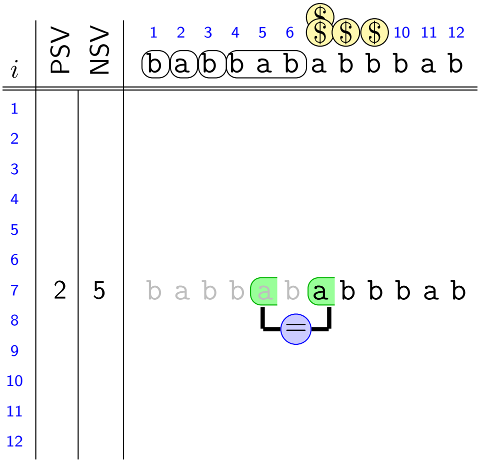

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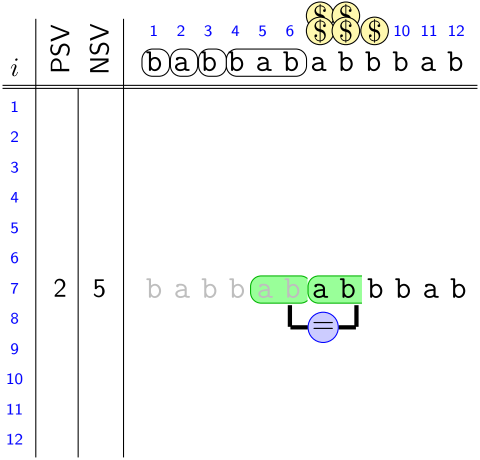

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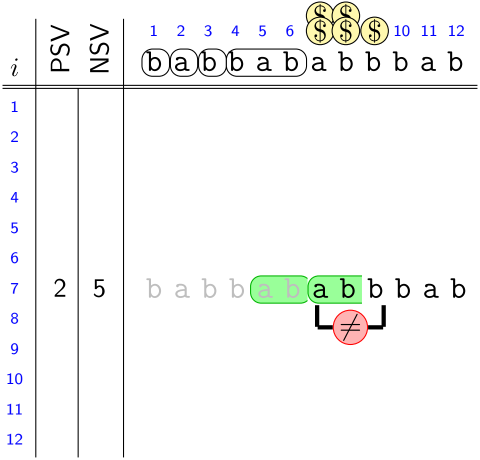

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2n log n algorithm

Goal

Space efficient simulation of all p_i values.

i	PSV	NSV	b a b b a b a b b b a b
1			
2			
3			
4			
5			
6			
7	2	5	b a b b a b a b b b a b
8			
9			
10			
11			
12			

LZ77: (b,0),(a,0),(1,1),(1,3)

2n log n algorithm

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Space efficient simulation of all p_i values.

i	PSV	NSV	1 2 3 4 5 6 $ $ $ 10 11 12 b a b b a b a b b b a b
1			
2			
3			
4			
5			
6			
7	2	5	b a b b a b a b b b a b
8			
9			
10			
11			
12			

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

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i	PSV	NSV	b a b b a b a b b b a b
1			
2			
3			
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5			
6			
7	2	5	b a b b a b a b b b a b
8			
9			
10			
11			
12			

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

2n log n algorithm

Goal

Space efficient simulation of all p_i values.

Fact

Given PSV and NSV arrays, LZ77 parsing can be computed in linear time.

			1 2 3 4 5 6 $ $ $ 10 11 12
i	PSV	NSV	(b)(a)(b)(b a b)(a b b)b a b
1			
2			
3			
4			
5			
6			
7	2	5	b a b b a b a b b b a b
8			
9			
10			
11			
12			

LZ77: (b,0),(a,0),(1,1),(1,3),(2,3)

Computing NSV/PSV arrays

- PSV/NSV can be computed from SA in linear time

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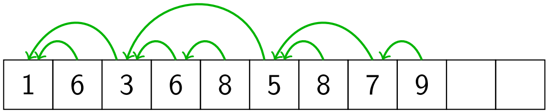

Computing NSV/PSV arrays

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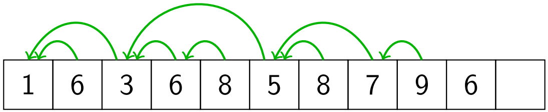
- no extra space required
- computation of PSV:



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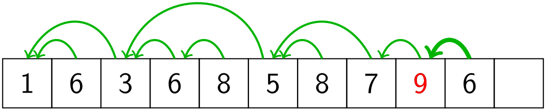

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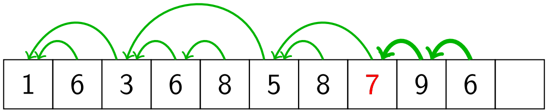

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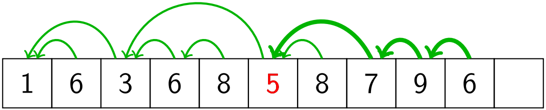

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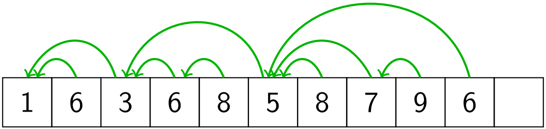

LZ77 with NSV/PSV arrays

Algorithm:

1. Compute SA

LZ77 with NSV/PSV arrays

Algorithm:

1. Compute SA
2. Compute PSV/NSV (in place)

LZ77 with NSV/PSV arrays

Algorithm:

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2. Compute PSV/NSV (in place)
3. Compute the factorization

LZ77 with NSV/PSV arrays

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   $\mathcal{O}(n)$ time

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Observation

We only need to access each PSV/NSV value *once*, in a left-to-right scan.

LZ77 with NSV/PSV arrays

Algorithm:

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   $\mathcal{O}(n)$ time, $3n \log n$ bits of extra space

Observation

We only need to access each PSV/NSV value *once*, in a left-to-right scan.

Lemma

A scan of NSV and PSV can be simulated with only one of them. It takes linear time and requires no extra space.

Simulating the scan of NSV/PSV

- Goal: at step i know the value of PSV$[i]$ and NSV$[i]$.

0	13	1	9	2	3	11	8	10	4	12	6	15	7	14	5	16	0

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- Goal: at step i know the value of PSV$[i]$ and NSV$[i]$.

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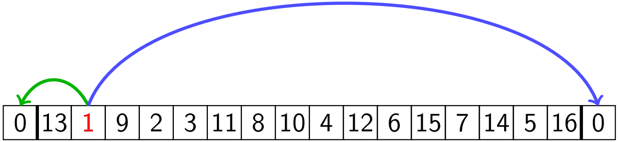

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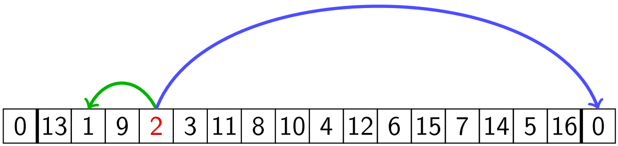

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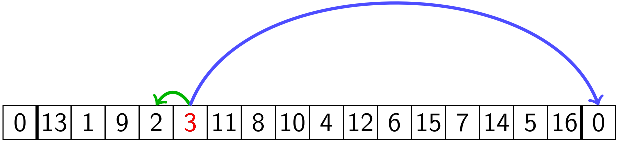

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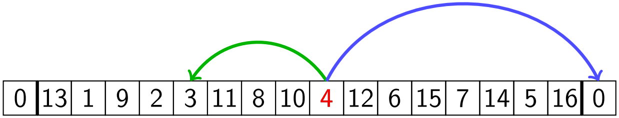

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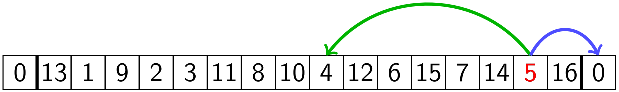

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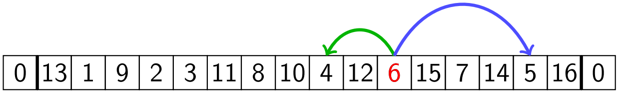

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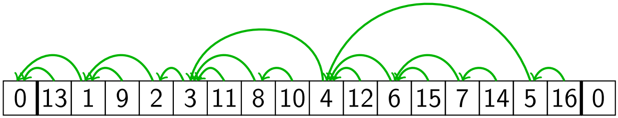

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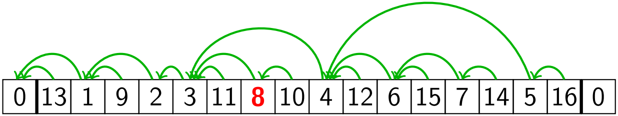

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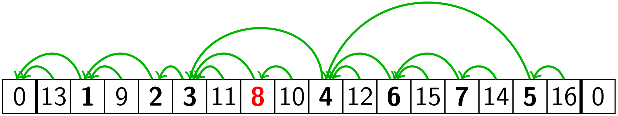

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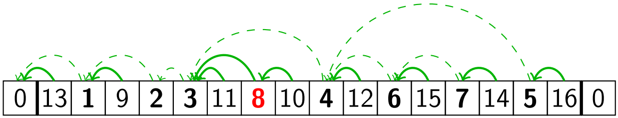

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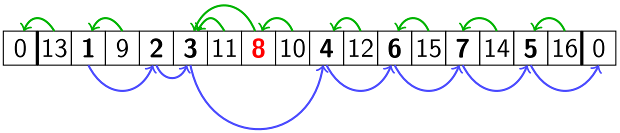
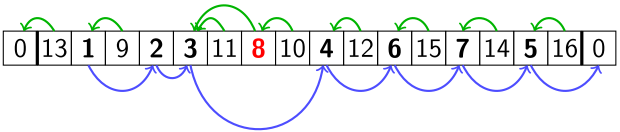


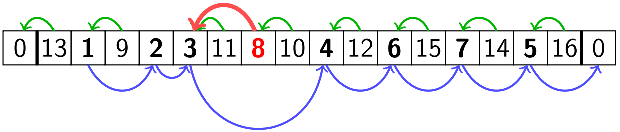
Simulating the scan of NSV/PSV

- Goal: at step i know the value of PSV$[i]$ and NSV$[i]$.
- PSV$[8] =$



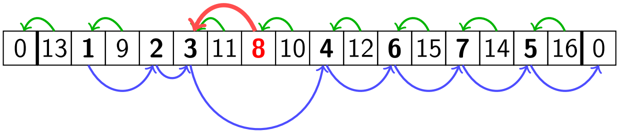
Simulating the scan of NSV/PSV

- Goal: at step i know the value of PSV$[i]$ and NSV$[i]$.
- PSV$[8] = 3$



Simulating the scan of NSV/PSV

- Goal: at step i know the value of PSV$[i]$ and NSV$[i]$.
- PSV$[8] = 3$
- NSV$[8] =$



Simulating the scan of NSV/PSV

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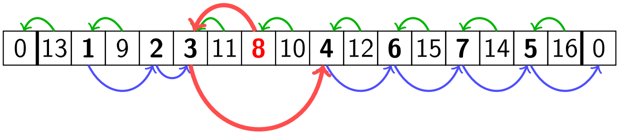
- NSV$[8] = 4$



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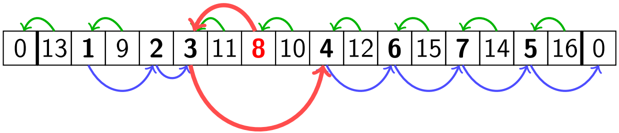
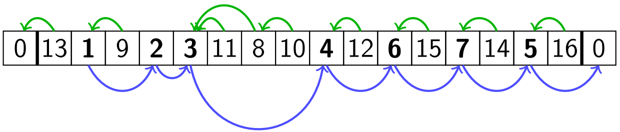


Simulating the scan of NSV/PSV

- Goal: at step i know the value of PSV$[i]$ and NSV$[i]$.
- PSV$[8] = 3$
- NSV$[8] = 4$
- Updating the links:



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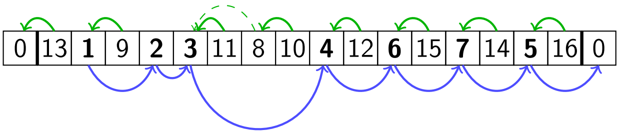

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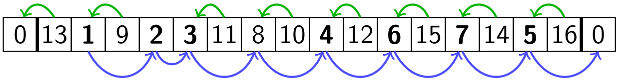

Simulating the scan of NSV/PSV

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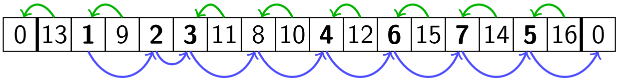
- PSV$[8] = 3$
- NSV$[8] = 4$
- Updating the links: $\mathcal{O}(1)$ time



The end

Thank you!

Experiments

- Dataset from Pizza & Chilli corpus

Experiments

- Dataset from Pizza & Chilli corpus
- Measured the time to compute LZ factorization

Experiments

- Dataset from Pizza & Chilli corpus
- Measured the time to compute LZ factorization
  - we exclude the time to compute SA

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Alg.	Mem	pro	eng	dna	src	cor	cer	ker	ein	tm29
K3	$13n$	74.5	75.7	81.7	50.5	43.6	63.2	45.7	56.9	38.2
K2	$9n$	84.1	80.6	92.7	54.8	40.2	53.2	41.5	43.5	35.1
ISA6r	$6n$	-	-	-	-	43.3	51.8	39.2	31.1	34.2
ISA6s	$6n$	198.0	171.0	175.2	115.0	49.4	56.3	45.7	37.1	39.6
ISA9	$9n$	92.7	83.9	86.1	59.3	41.9	53.0	42.8	45.2	36.4
iBGS	$17n$	99.8	93.2	97.5	69.3	51.5	65.5	52.9	60.0	44.1
iBGL	$17n$	123.2	108.6	113.4	77.8	52.2	66.1	53.0	58.6	44.2
iBGT	$13n$	171.4	153.9	188.0	99.8	55.4	84.1	56.2	52.8	44.4