Compressed String Dictionary Look-up with Edit Distance One

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Problem

- Design a (compressed) index for a set $D=\{S_1, S_2, ..., S_d\}$ of d strings of total length n drawn from an alphabet of size σ
- Given any pattern P, report all the occ strings in D having Edit Distance at most one from P
  - edit(P, S) = minimum # of edit operations (insertion, deletion, substitution) to transform P into S

Motivations

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- Information Retrieval

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  - e.g., users may misspell terms in their queries
- Database
  - e.g., you may do not remember the correct spelling of a key
- Data Mining
  - e.g, clean a noised dataset

Known solutions

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- G. Brodal and L. Gasieniec [CPM'96]
  - Space: O(σn log n) bits
  - Time: O(|P| + occ)

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Reduced to membership queries
in a super set containing
all the strings that have
edit distance one from any string in D.

For each S∈D, we add O(σ|S|)

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Use Balanced Search Trees to index these factorizations

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Similar approach

Use of Minimal Perfect
Hash Functions to index factorizations
and to save space

Our results

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Solution based on Patricia tries and Perfect hashing

Time $O(|P| + occ)$

Space $2nH_k + n \cdot o(\log \sigma) + 2d \log d$

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Solution based on Compressed Permuterm index

Time $O(|P| \log \log \sigma + occ)$

Space $nH_k + n \cdot o(\log \sigma)$ for $\sigma = \omega(\log^c n)$

Our results

Solution based on Patricia tries and Perfect hashing

Time $O(|P| + occ)$

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Top-k strings in $O(|P| + k \log k)$

Solution based on Compressed Permuterm index

Time $O(|P| \log \log \sigma + occ)$

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access time: O(1)
space: H_k + n o(log σ)

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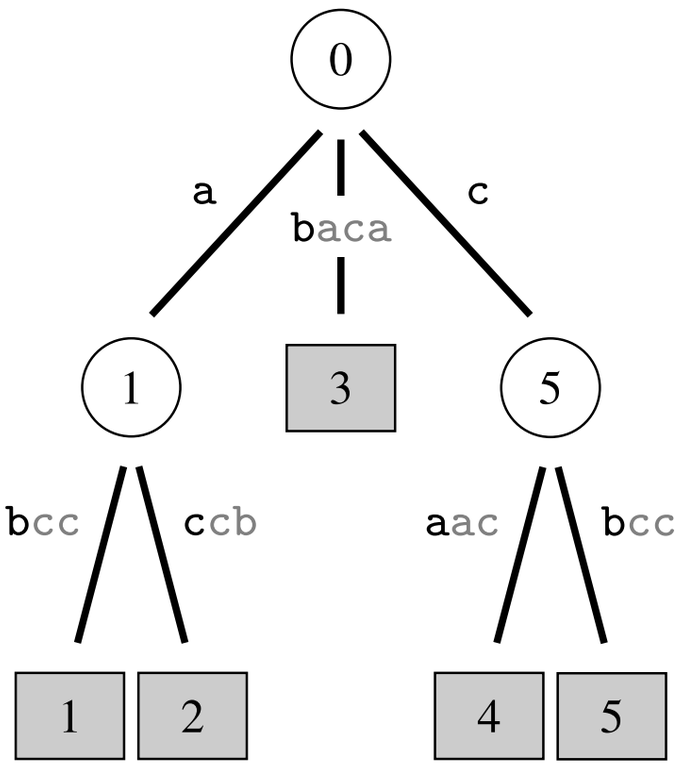

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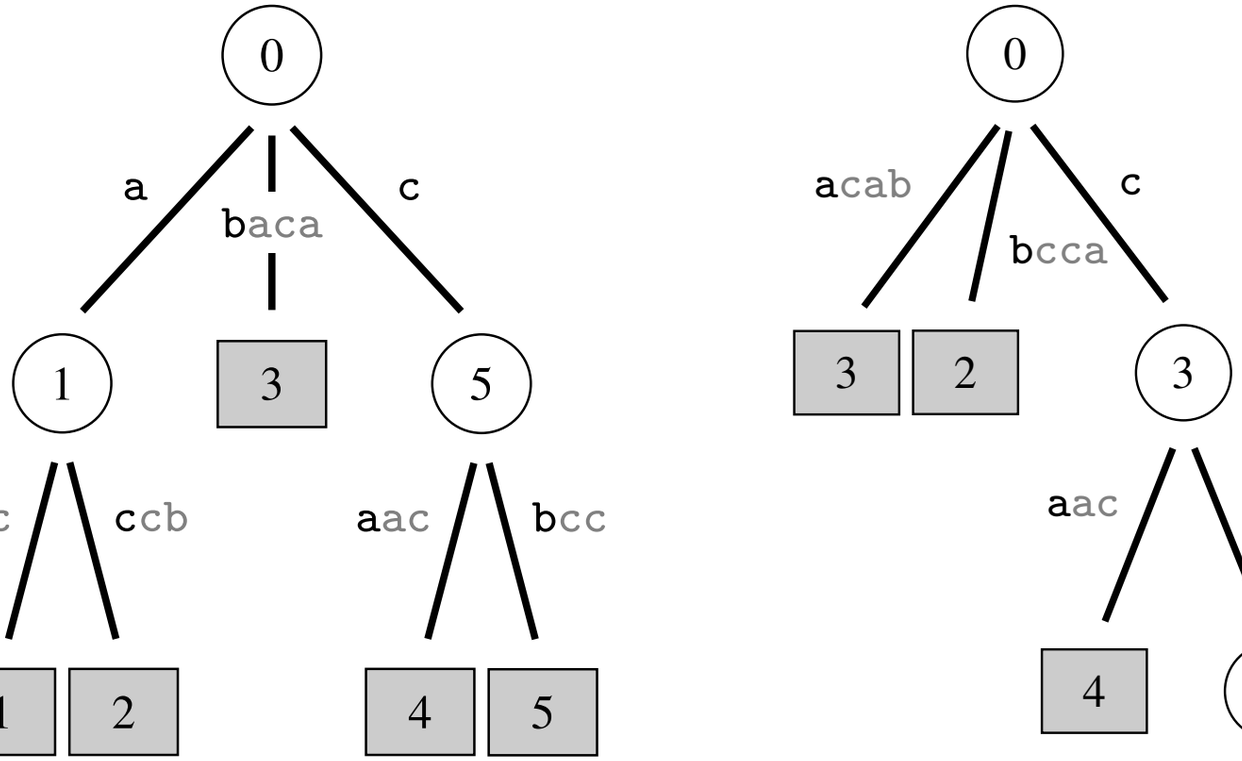

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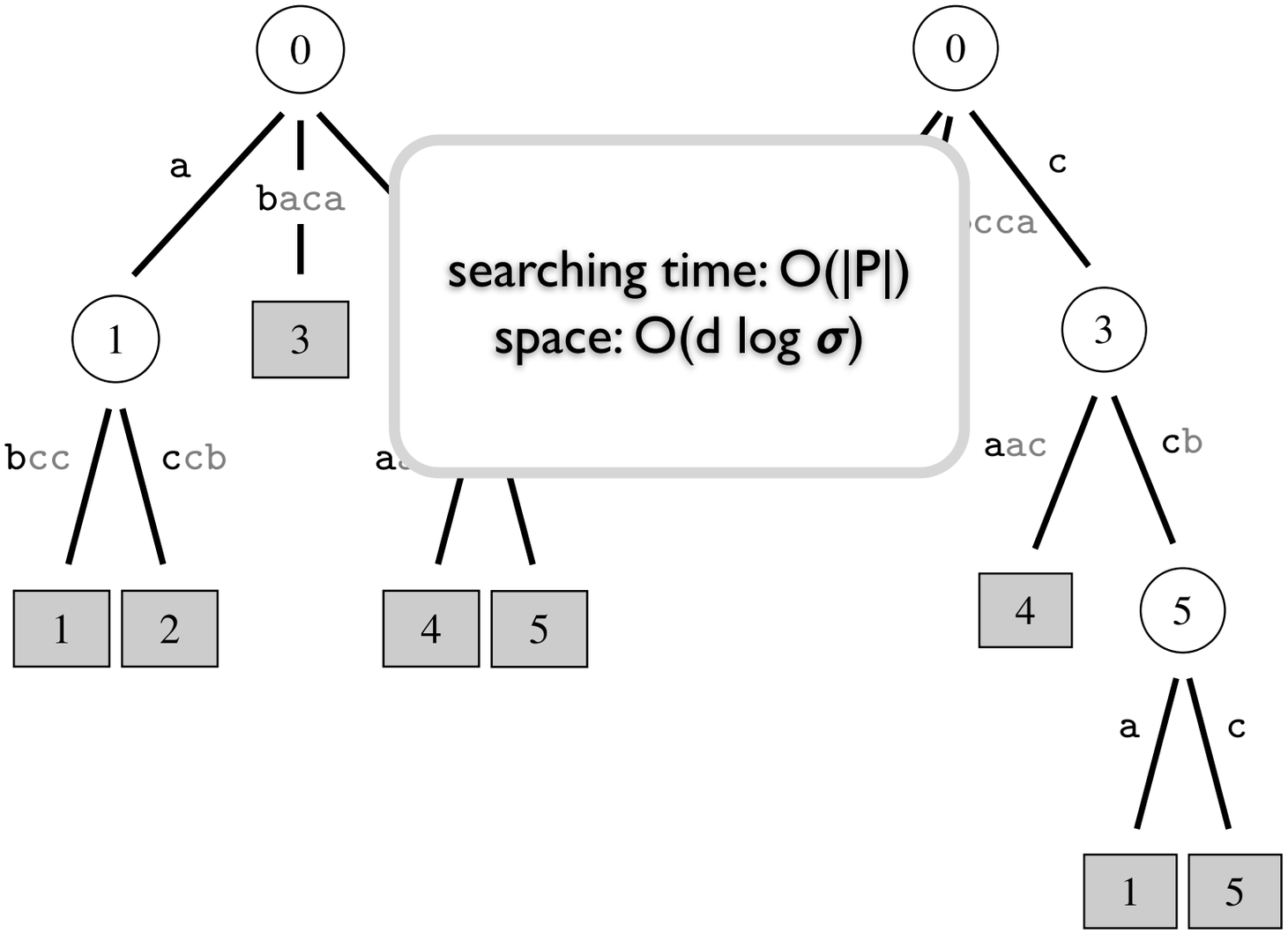

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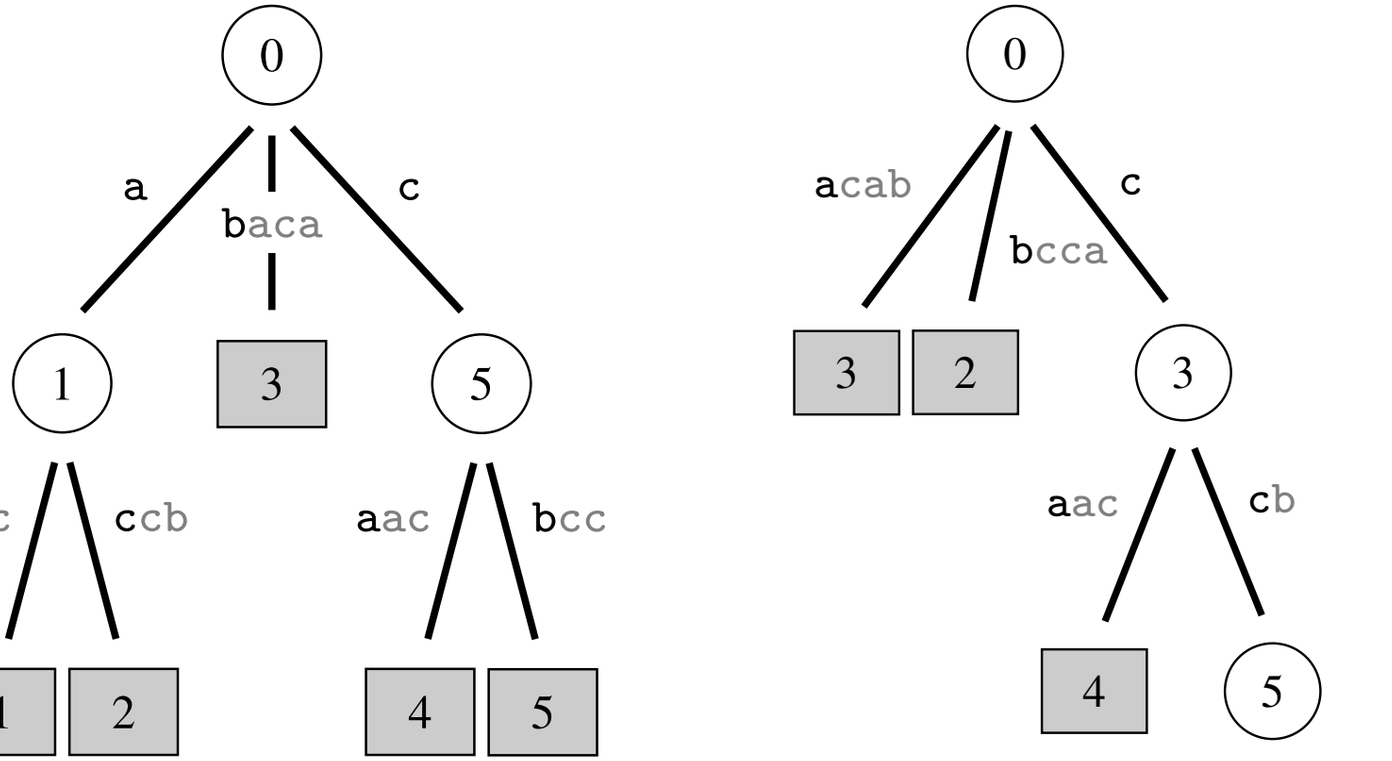

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lcp	1	0	0	1	×
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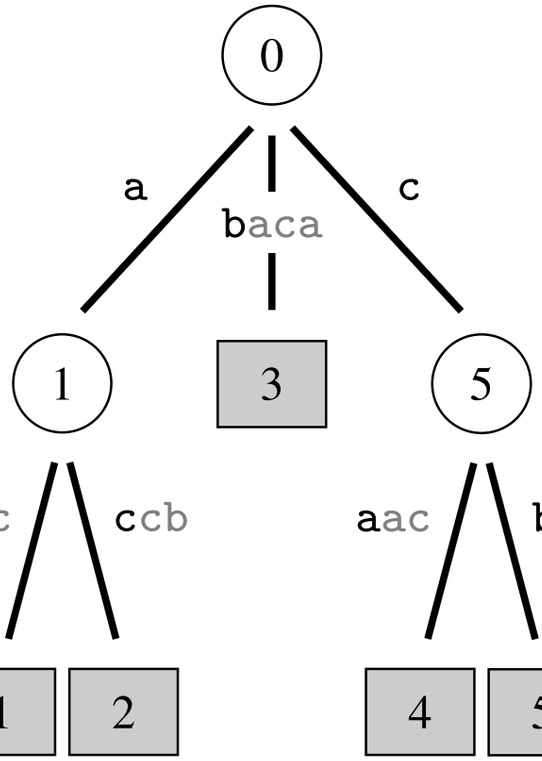

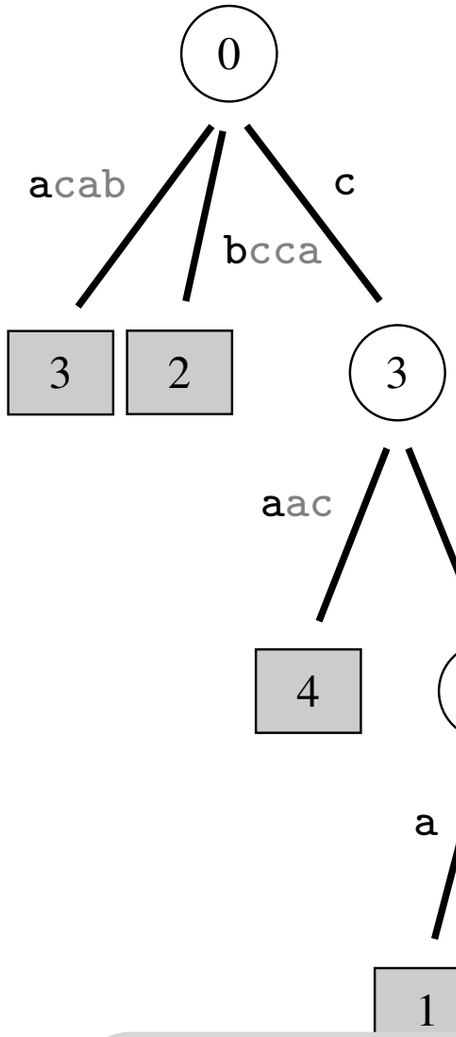

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any LCP: O(1)
space:
O(d (log d + log (n/d))

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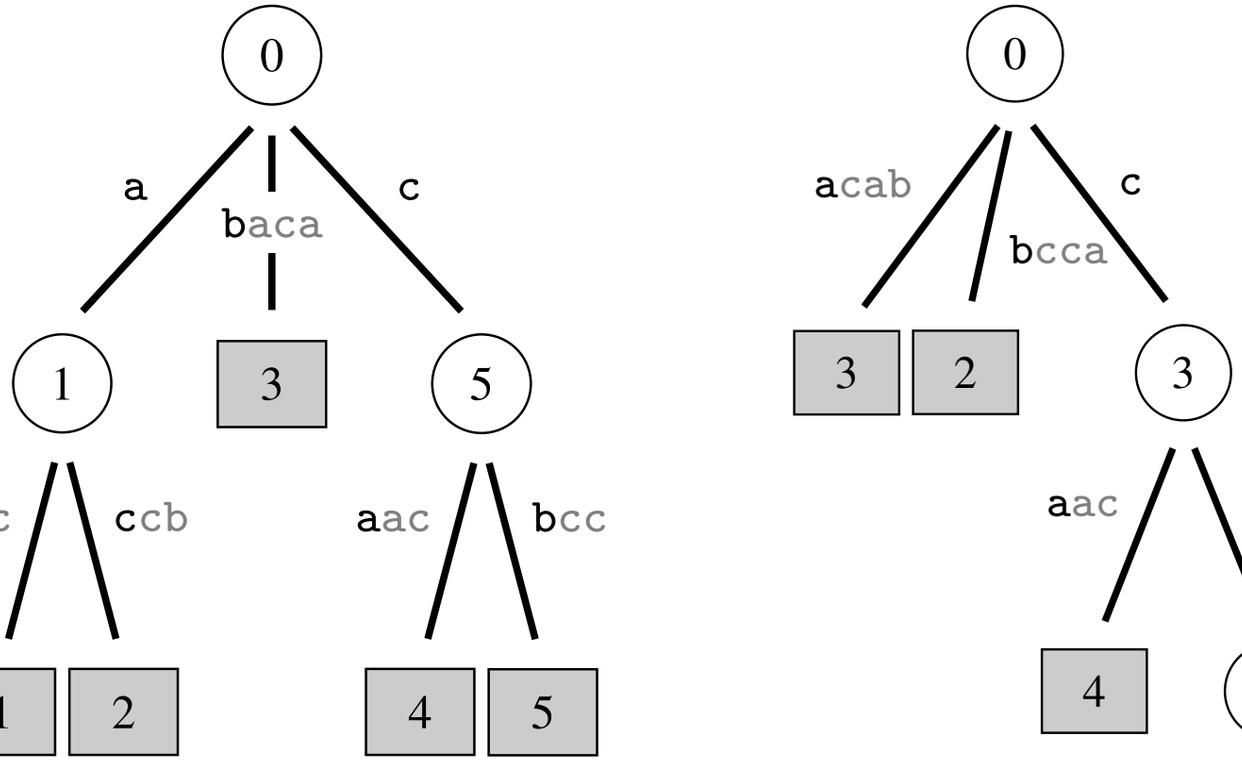

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Factorizations
of the form

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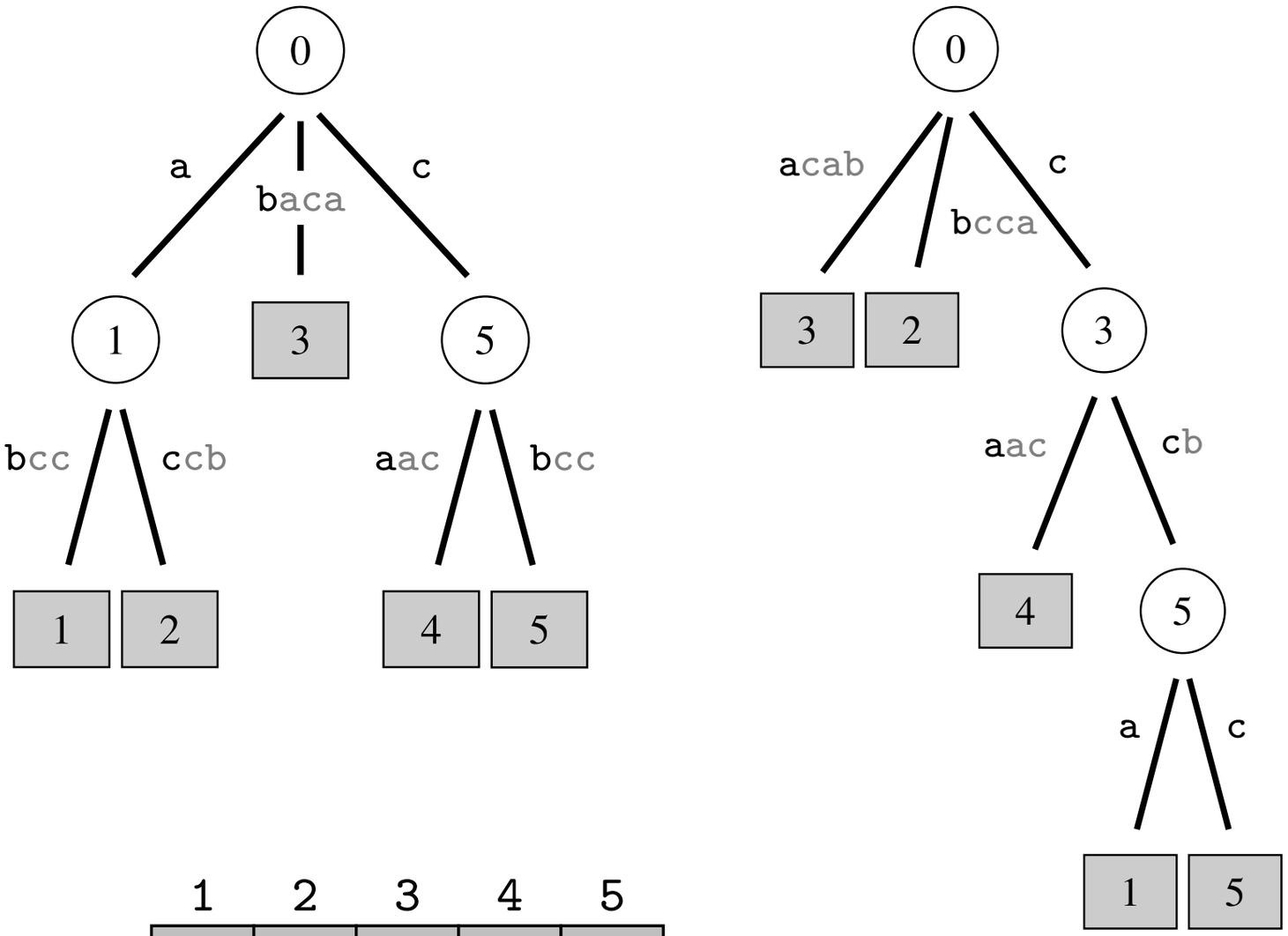

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a b c c

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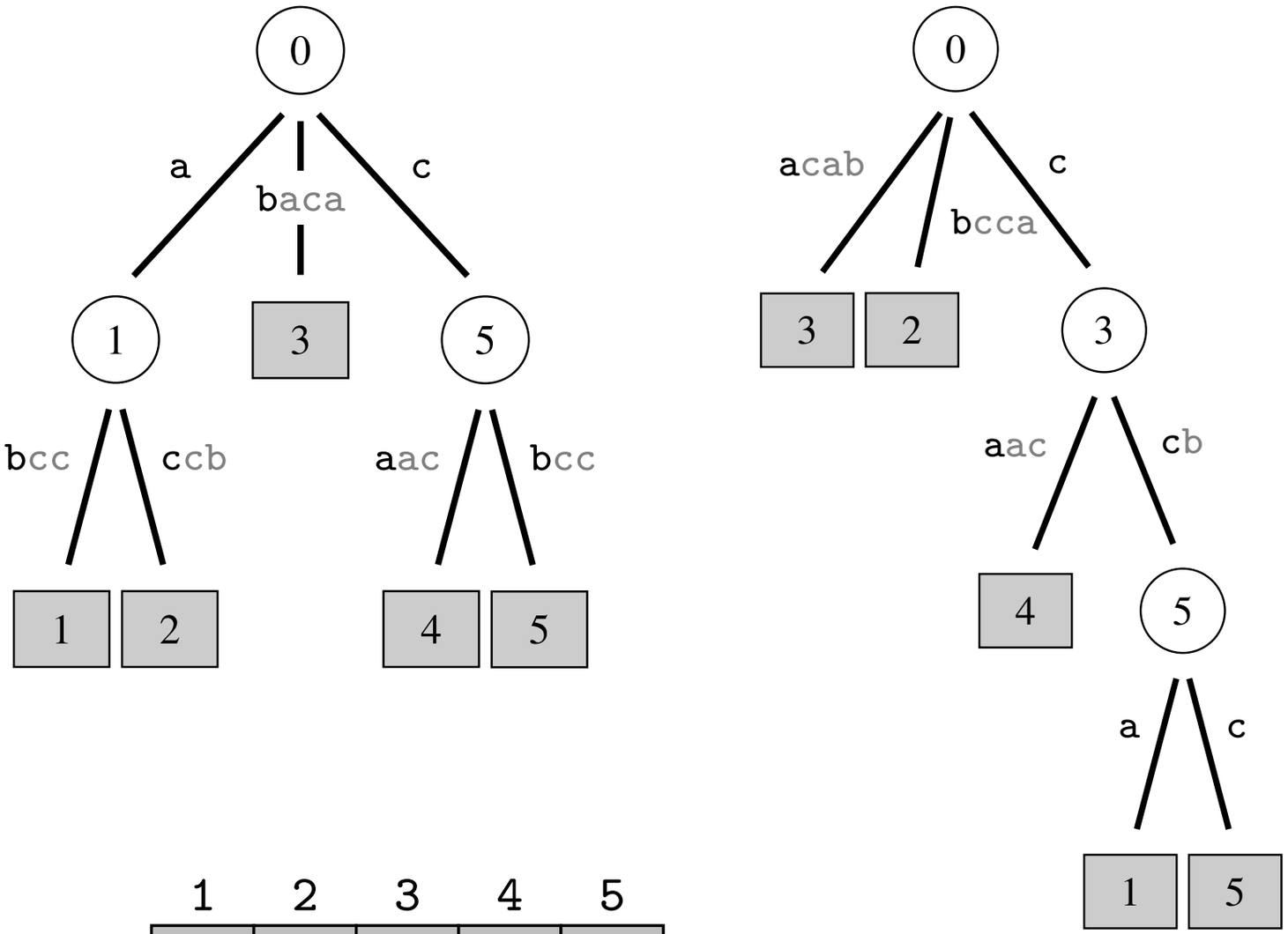

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S[1,i-1] in PT

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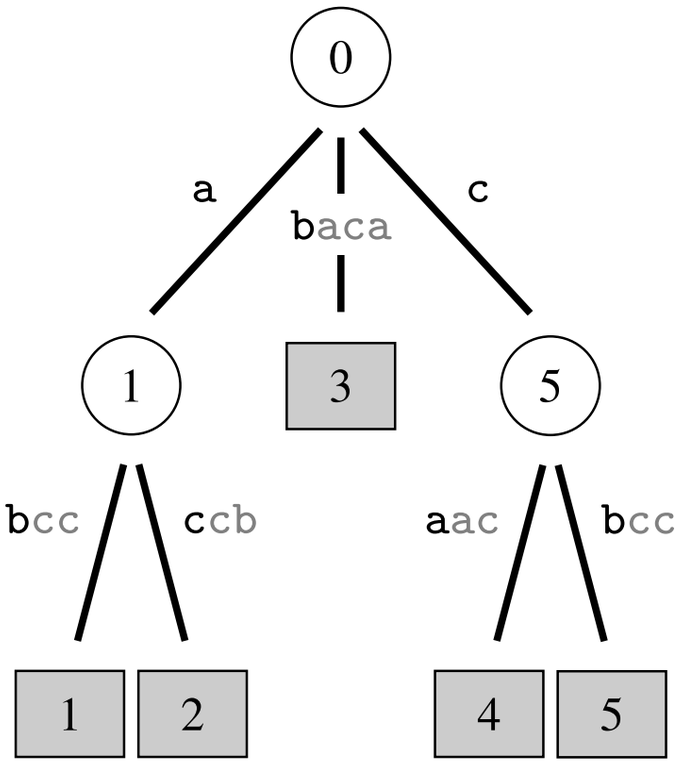

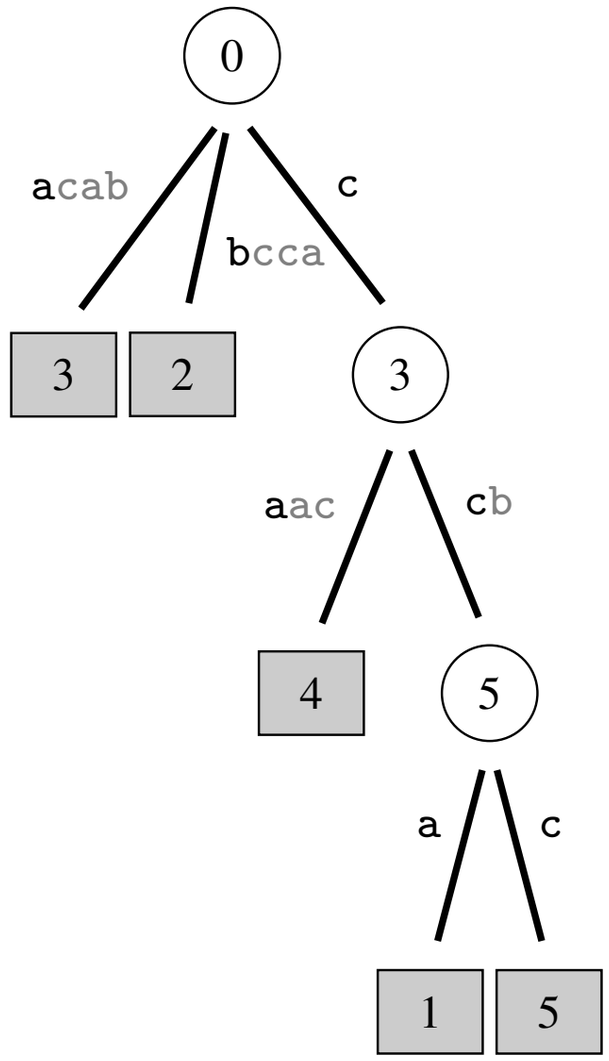

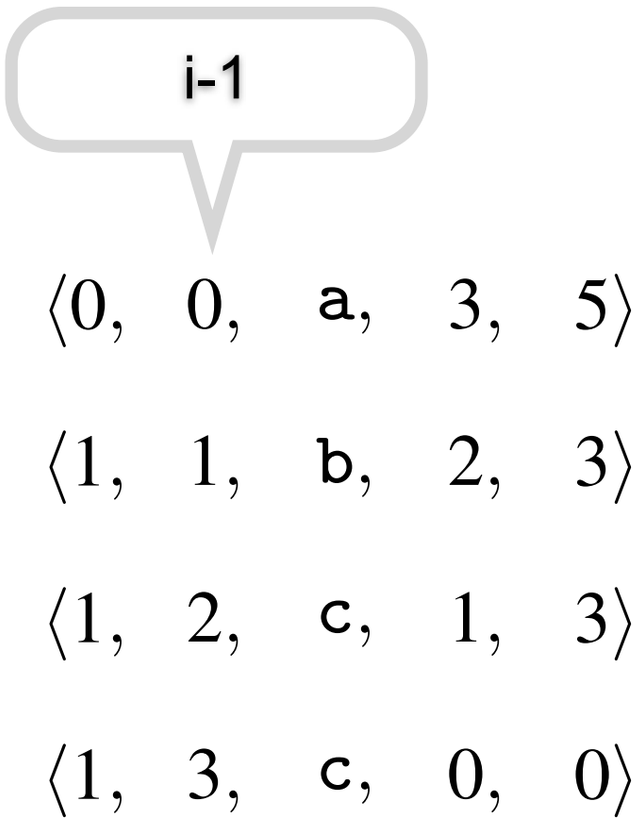

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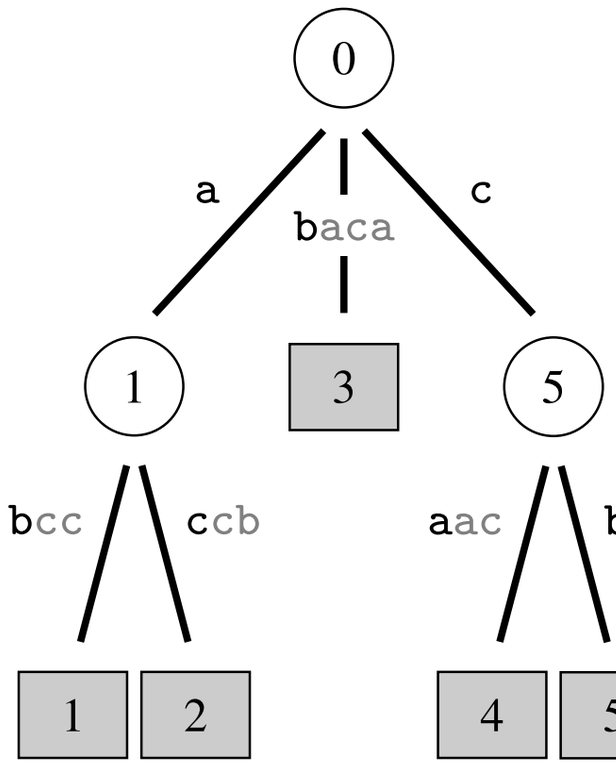

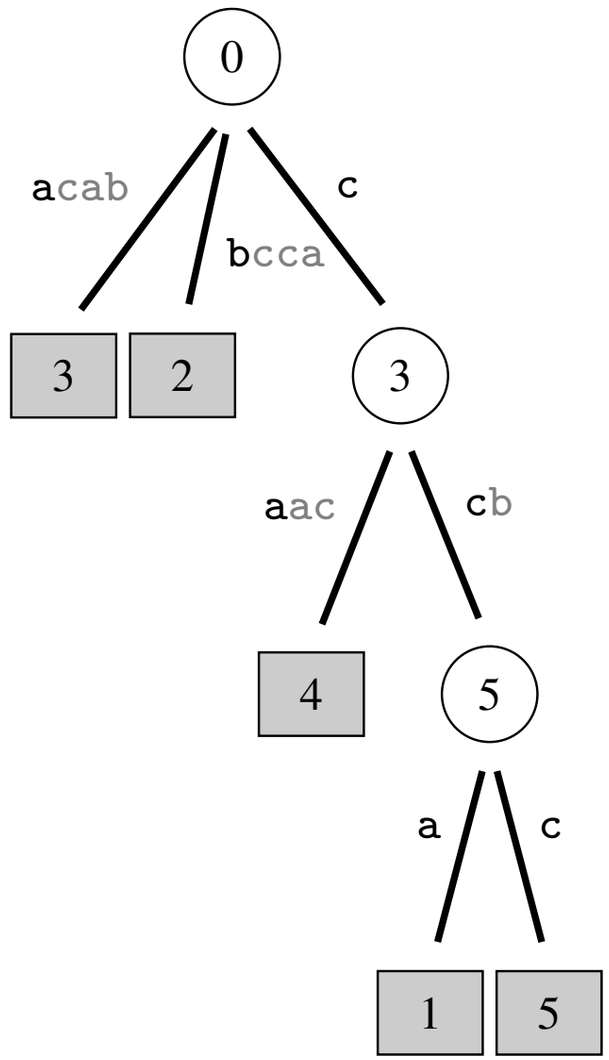

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S[i]

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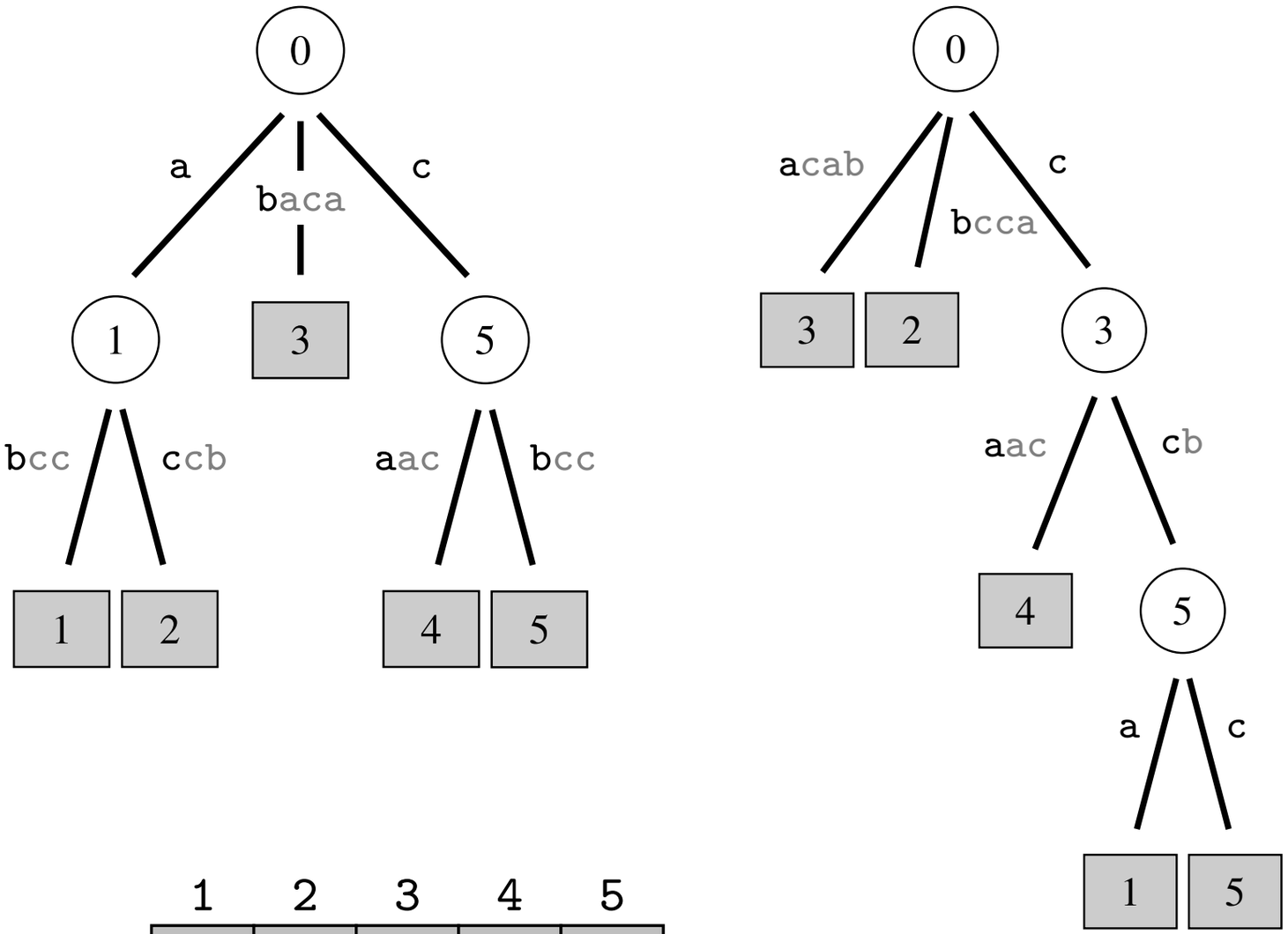

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S[i+1,s] in PT^r

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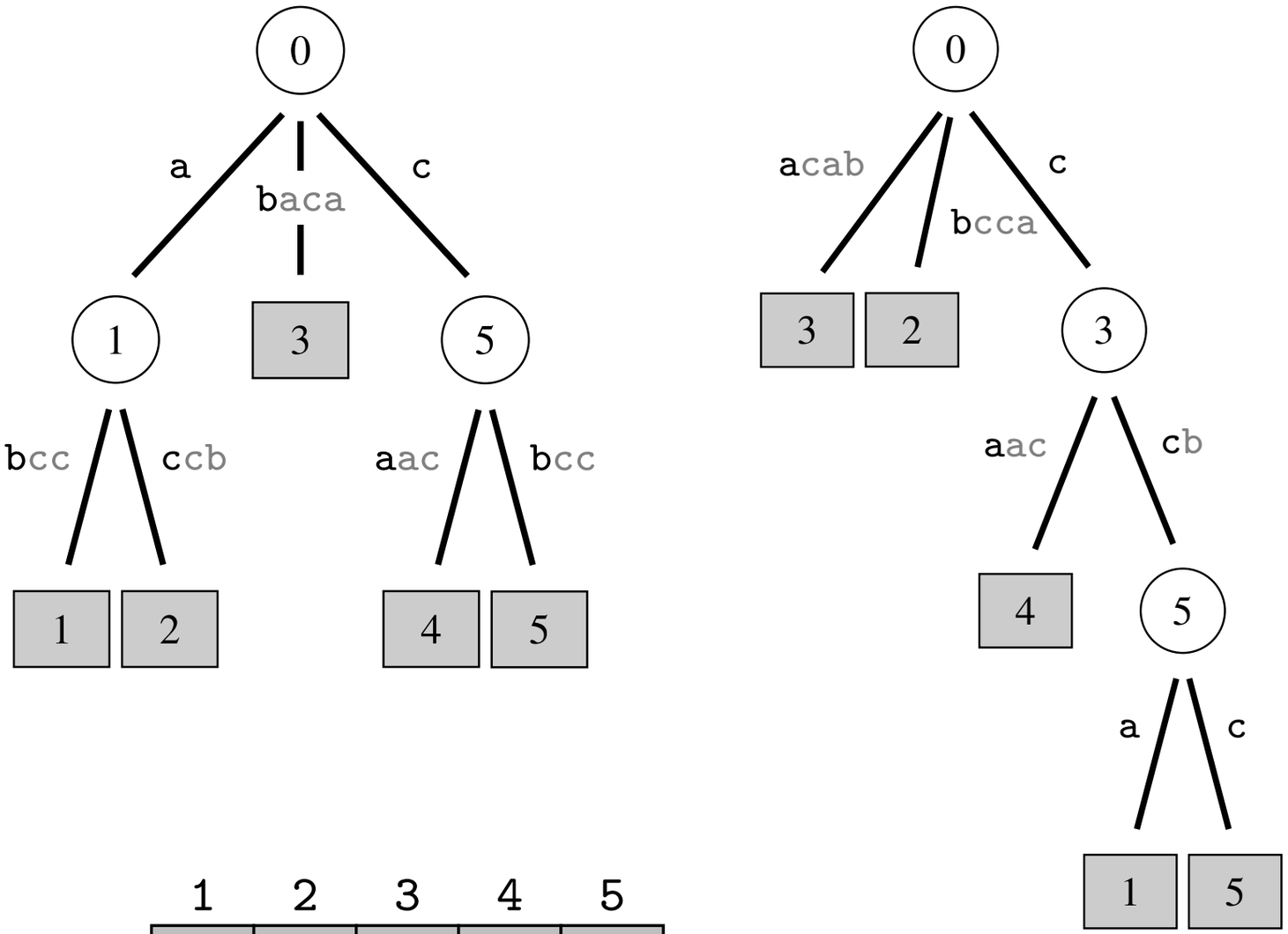

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s-i-1

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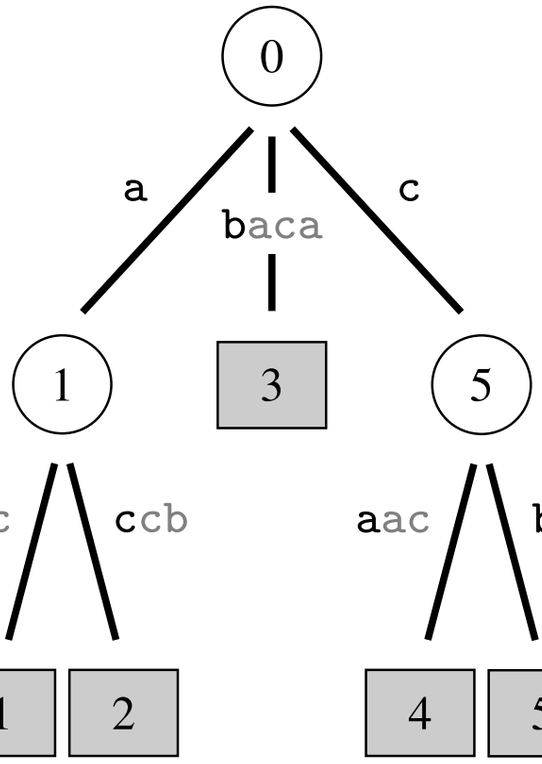

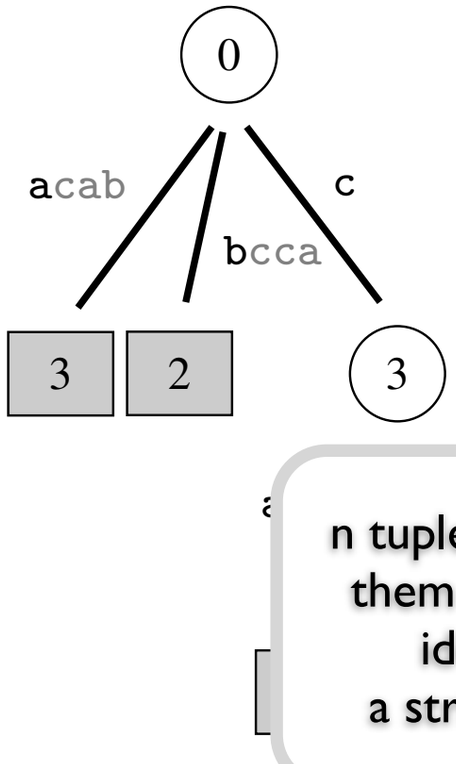

n tuples. Each of them uniquely identify a string in D

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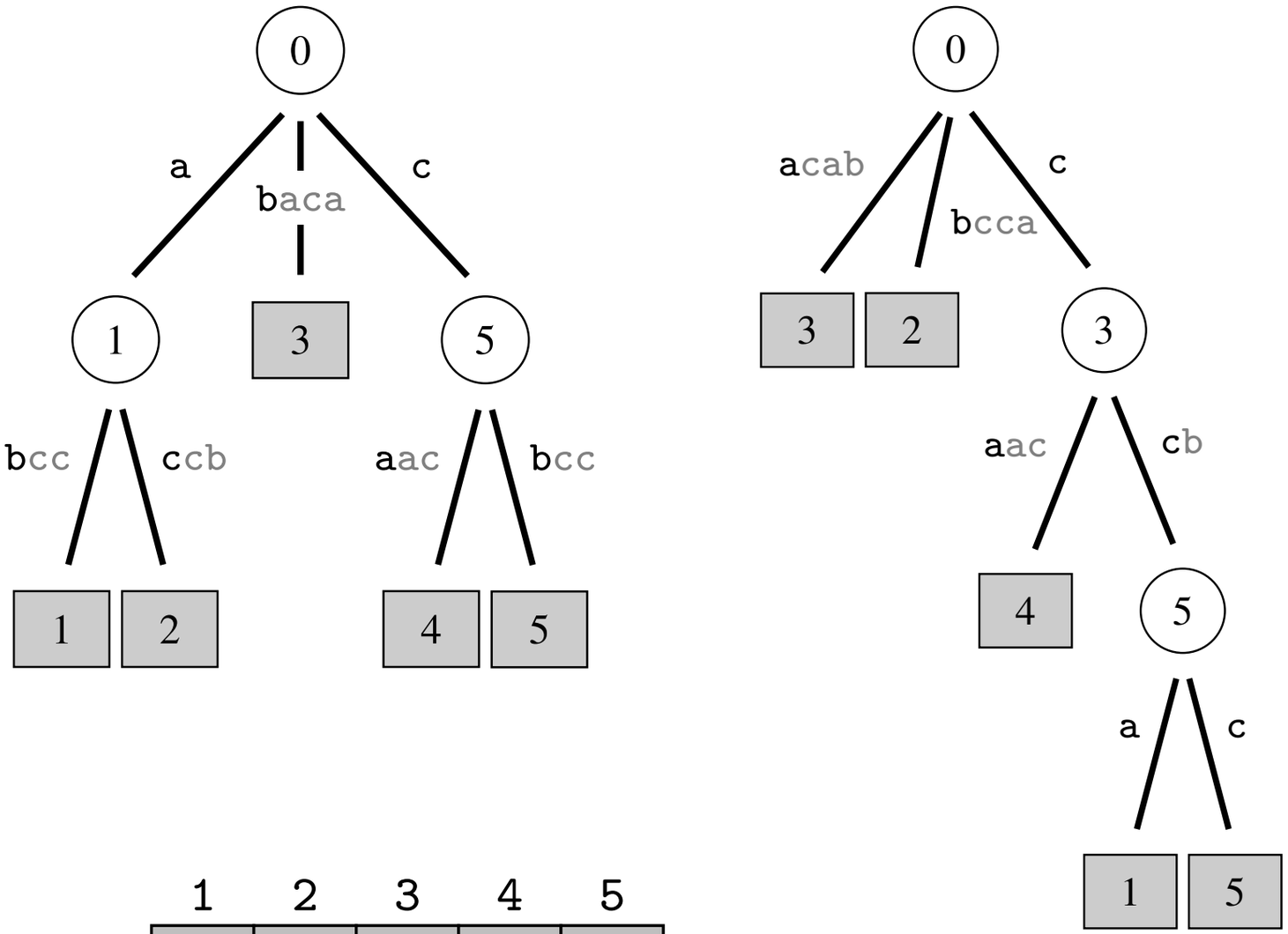

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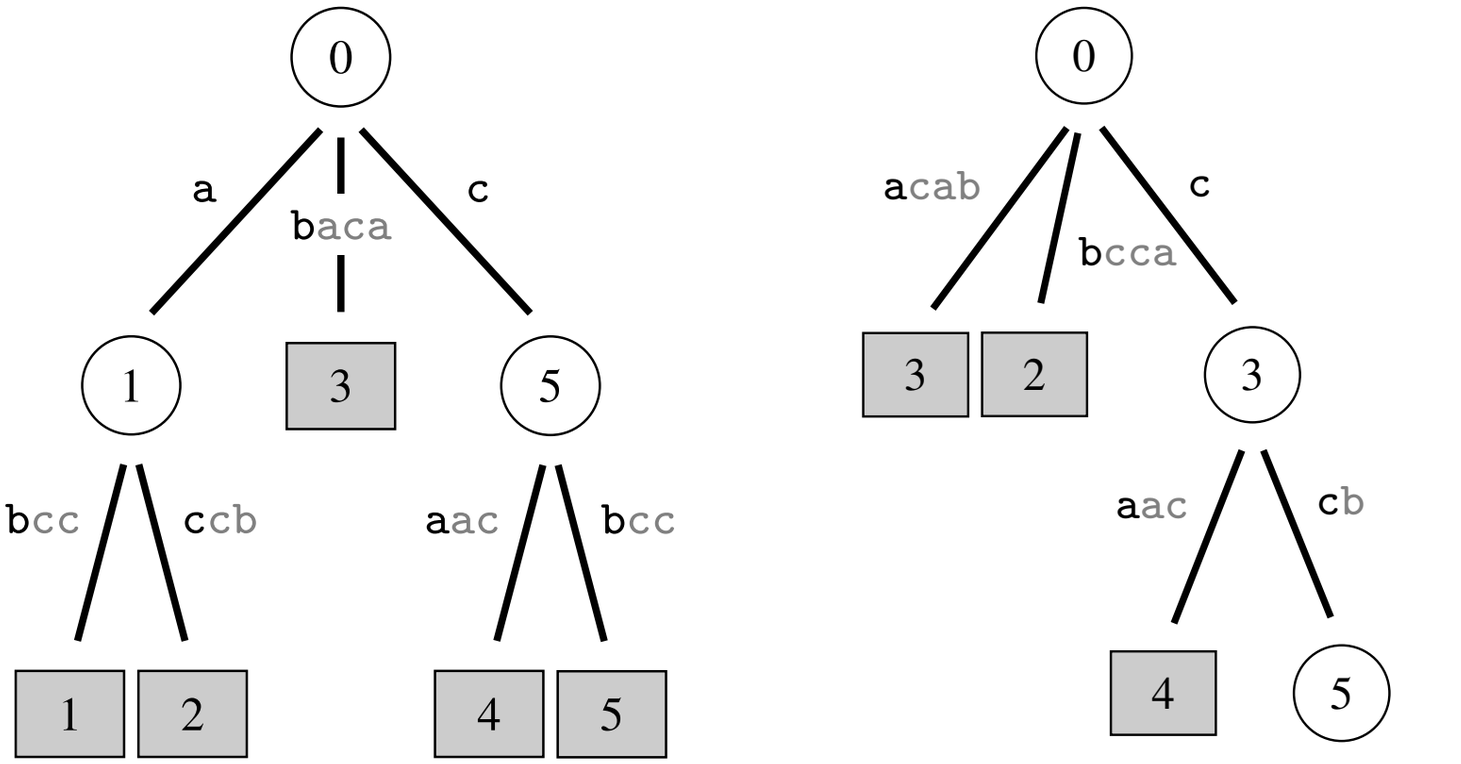
Find insertions in P[1,p]

Search factorizations P[1,i] P[i+1,p]

np_i, i sp_i, p-i+1



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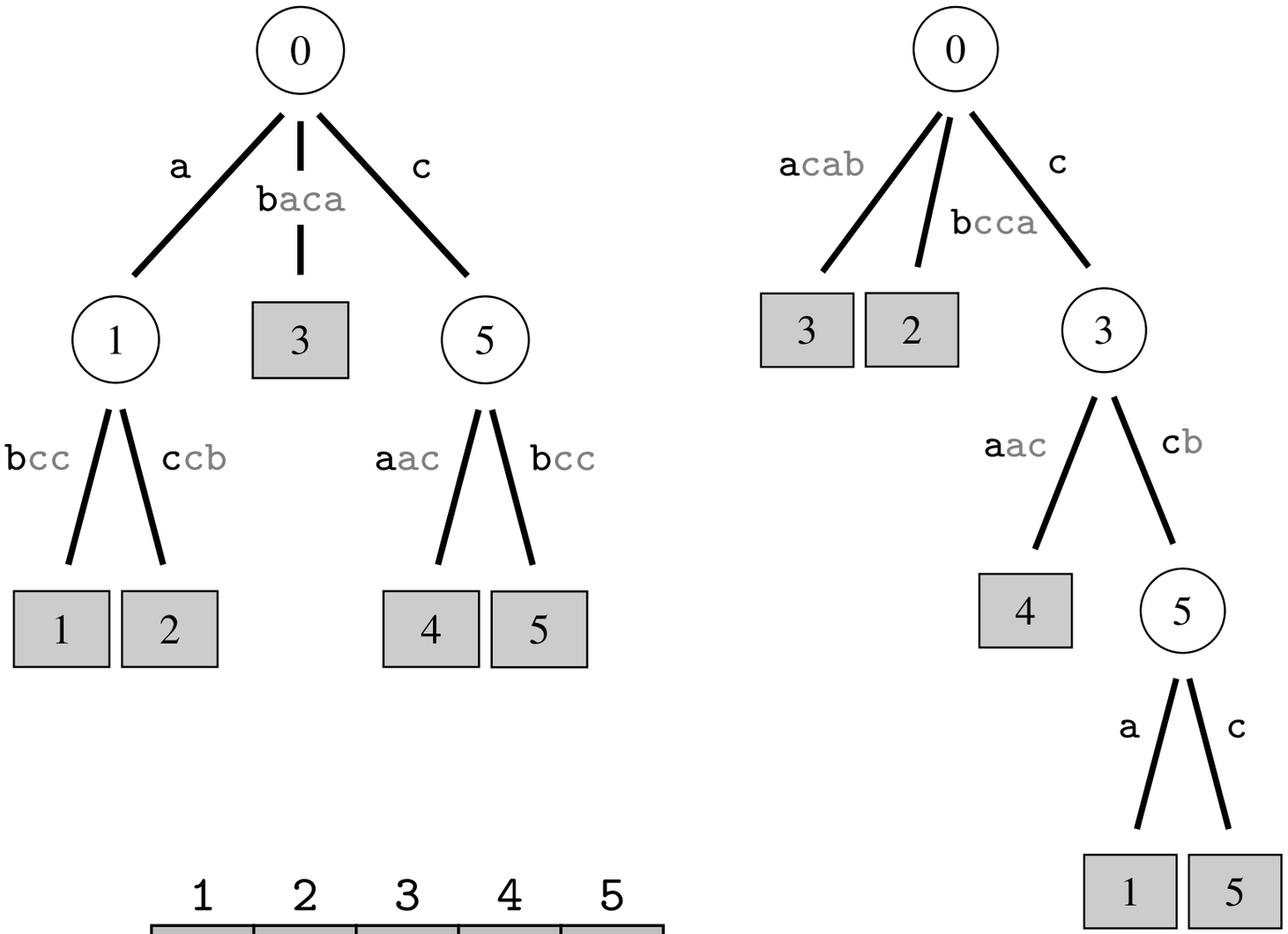

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All in O(p) time

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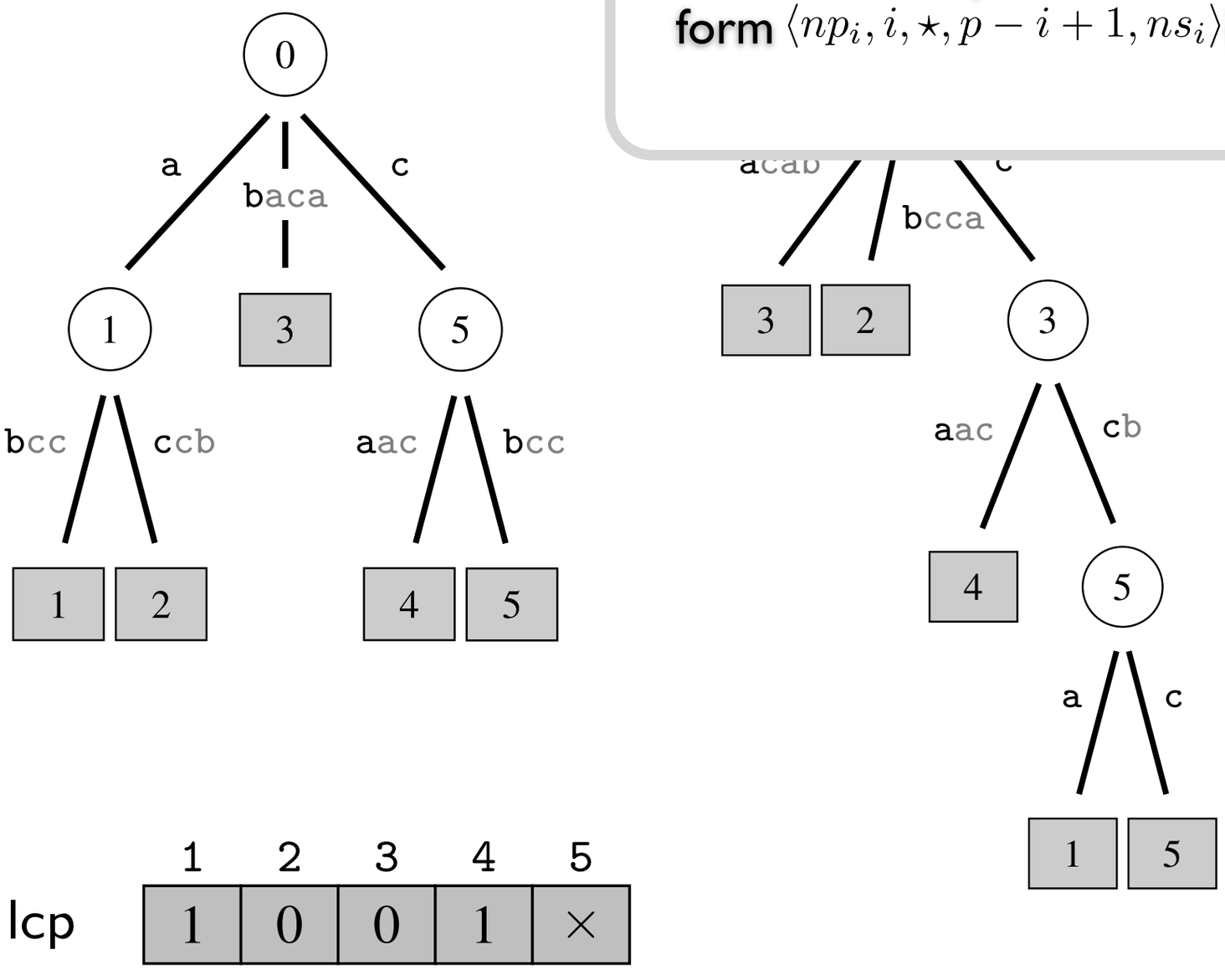
a b c c

$$D = \{\texttt{abcc}, \texttt{accb}, \texttt{baca}, \texttt{caac}, \texttt{cbcc}\}$$

$$cmp(D) = \texttt{abcc ac}$$

Are there tuples of the form $\langle np_i, i, \star, p - i + 1, ns_i \rangle$?



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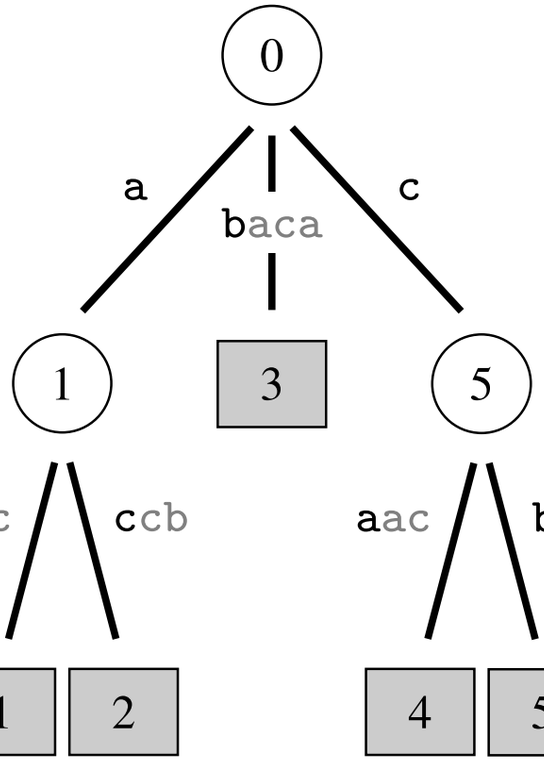
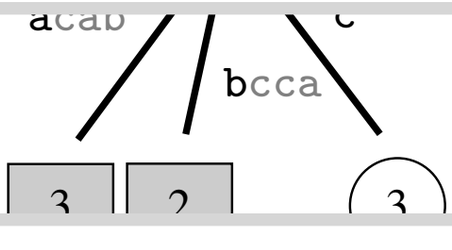


Are there tuples of the form $\langle np_i, i, \star, p-i+1, ns_i \rangle$?

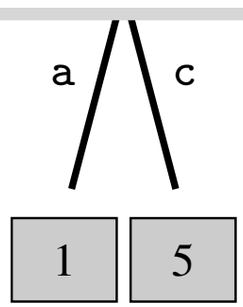


Yes?

$P[1,i] \star P[i+1,p]$

is in D

(and has distance one from P)



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Search factorizations P[1,i] P[i+1,p]

np_i, i $\qquad$ sp_i, p-i+1

All in O(p) time

$\langle 0, \quad 0, \quad \texttt{a}, \quad 3, \quad 5 \rangle$

$\langle 1, \quad 1, \quad \texttt{b}, \quad 2, \quad 3 \rangle$

$\langle 1, \quad 2, \quad \texttt{c}, \quad 1, \quad 3 \rangle$

$\langle 1, \quad 3, \quad \texttt{c}, \quad 0, \quad 0 \rangle$

a b c c

	1	2	3	4	5
lcp	1	0	0	1	×
lcp_r	0	0	1	3	×
π	3	2	4	1	5

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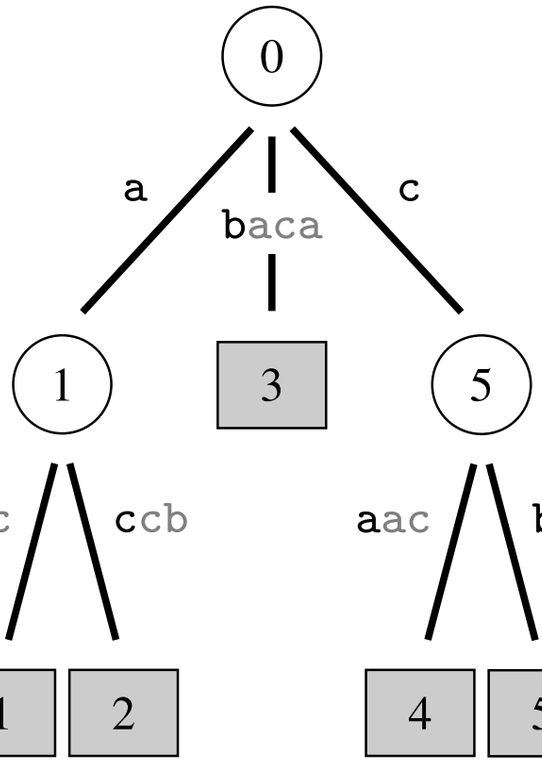

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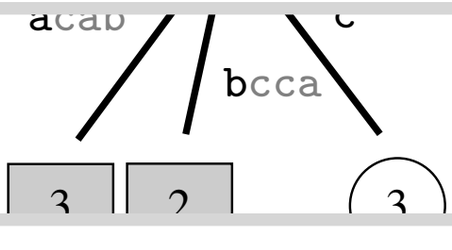

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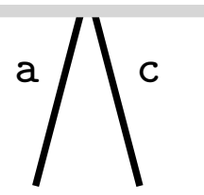

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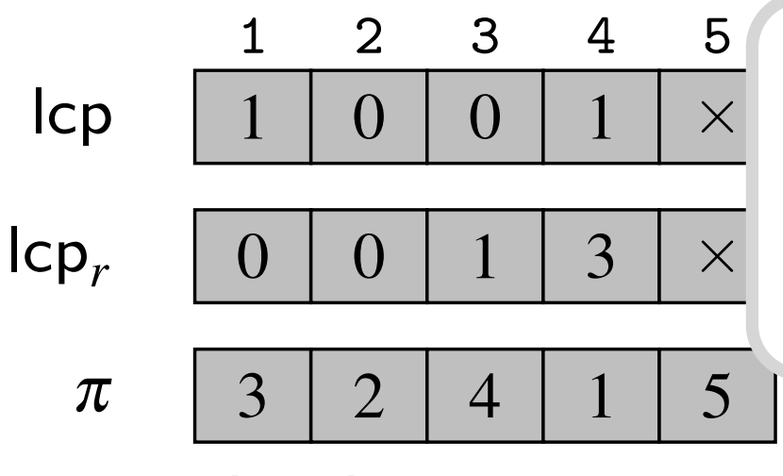
$\langle 1, \quad 3, \quad \texttt{c}, \quad 0, \quad 0 \rangle$

a b c c



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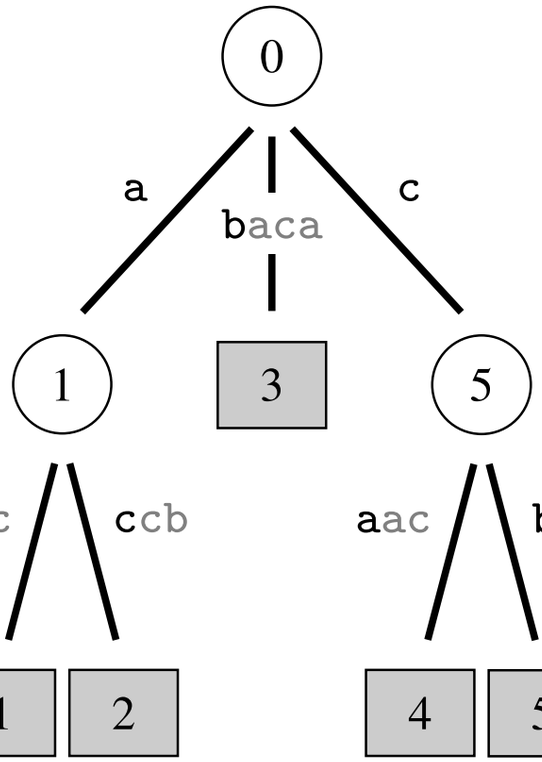

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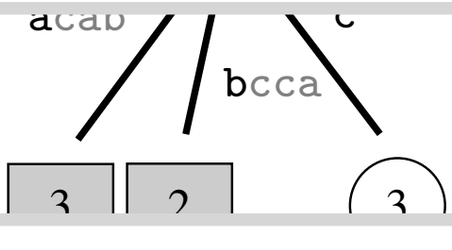

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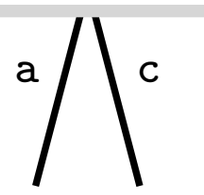

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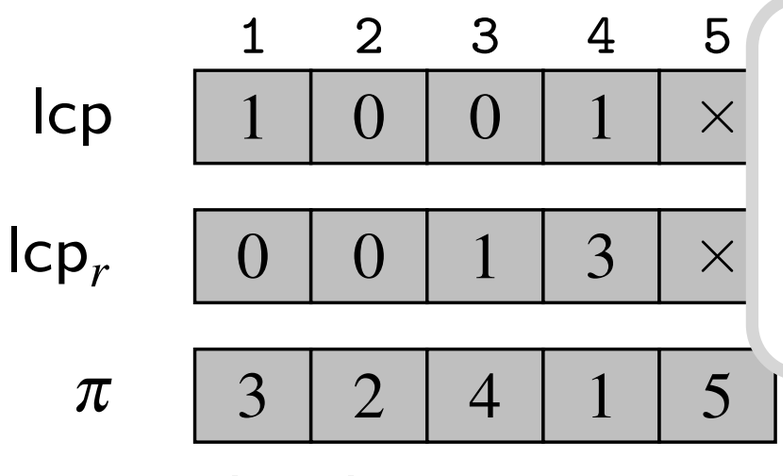
$\langle 1, \quad 3, \quad \texttt{c}, \quad 0, \quad 0 \rangle$

a b c c



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a b c c

$$D = \{\texttt{abcc}, \texttt{accb}, \texttt{baca}, \texttt{caac}, \texttt{cbcc}\}$$

Define the function F, so that,

$$F(np, i, j, sp) = c$$

iff there exists tuple

$$\langle np, i, \mathbf{c}, j, sp \rangle$$

$\langle 0, \quad 0, \quad \texttt{a}, \quad 3, \quad 5 \rangle$

$\langle 1, \quad 1, \quad \texttt{b}, \quad 2, \quad 3 \rangle$

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to an alphabet of size σ can be represented within
$nH_0 + n\, o(H_0)$ bits, so that
if x ∈ S, F(x) evaluated in constant time
if x ∉ S, an arbitrary symbol is returned

$$\langle 0, \quad 0, \quad \texttt{a}, \quad 3, \quad 5 \rangle$$

$$\langle 1, \quad 1, \quad \texttt{b}, \quad 2, \quad 3 \rangle$$

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H_k is achieved by splitting F into several functions based on k-long context preceding the symbol

$$\langle 0, \quad 0, \quad \texttt{a}, \quad 3, \quad 5 \rangle$$

$$\texttt{b}, \quad 2, \quad 3 \rangle$$

$$\texttt{c}, \quad 1, \quad 3 \rangle$$

$$\langle 1, \quad 3, \quad \texttt{c}, \quad 0, \quad 0 \rangle$$

a b c c

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We may err!
P[1,i] c P[i+1,p]
may not be in D.

We need to check
if it belongs to D!

tuples so that,
$p_i, i, sp_i, p - i + 1,$
n the symbols,
n tuples of the
$, \star, p - i + 1, ns_i \rangle$.

$\langle 0, \quad 0, \quad \texttt{a}, \quad 3, \quad 5 \rangle$

$\langle 1, \quad 1, \quad \texttt{b}, \quad 2, \quad 3 \rangle$

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in O(1).
see the paper

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a b c c

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iff there exists tuple

$$\langle np, i, \mathbf{c}, j, sp \rangle$$

Space: $2nH_k + n \cdot o(\log \sigma) + 2d \log d$

Time: $O(|P| + occ)$

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the

$ns_i\rangle$.

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$$\langle 1, \quad 3, \quad \texttt{c}, \quad 0, \quad 0 \rangle$$

a b c c

$D = \{\texttt{abcc}, \texttt{ac}$ ... $\texttt{cbcc}\}$

Define the fun

$F(np, i, j$

iff there e

$\langle np, i,$

Compressed Representation + Compressed function

LCP + LCS + identifiers

Space: $2nH_k + n \cdot o(\log \sigma) + 2d \log d$

Time: $O(|P| + occ)$

We may err!
P[1,i] c P[i+1,p]
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the

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a b c c

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Subset of strings having the same length p+1

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Subset of strings having the same length p+1

a b c c #
b c c # a
c c # a b
c # a b c
a b c c

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Subset of strings having the same length p+1

a b c c #
b c c # a
c c # a b
c # a b c
a b c c
a c c b #
c c b # a
c b # a c
b # a c c
a c c b
b a c a #
a c a # b
c a # b a
a # b a c
b a c a
c a a c #
a a c # c
a c # c a
c # c a a
c a a c
c b c c #
b c c # c
c c # c b
c # c b c
c b c c

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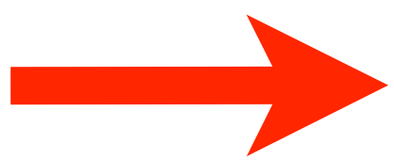
a b c c #
b c c # a
c c # a b
c # a b c
a b c c
a c c b #
c c b # a
c b # a c
b # a c c
a c c b
b a c a #
a c a # b
c a # b a
a # b a c
b a c a
c a a c #
a a c # c
a c # c a
c # c a a
c a a c
c b c c #
b c c # c
c c # c b
c # c b c
c b c c



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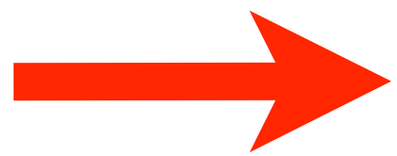
```
a b c c #
b c c # a
c c # a b
c # a b c
# a b c c
a c c b #
c c b # a
c b # a c
b # a c c
# a c c b
b a c a #
a c a # b
c a # b a
a # b a c
# b a c a
c a a c #
a a c # c
a c # c a
c # c a a
# c a a c
c b c c #
b c c # c
c c # c b
c # c b c
# c b c c
```



F				L
#	a	b	c	c
#	a	c	c	b
#	b	a	c	a
#	c	a	a	c
#	c	b	c	c
a	#	b	a	c
a	a	c	#	c
a	b	c	c	#
a	c	#	c	a
a	c	a	#	b
a	c	c	b	#
b	#	a	c	c
b	a	c	a	#
b	c	c	#	a
b	c	c	#	c
c	#	a	b	c
c	#	c	a	a
c	#	c	b	c
c	a	#	b	a
c	a	a	c	#
c	b	#	a	c
c	b	c	c	#
c	c	#	a	b
c	c	#	c	b
c	c	b	#	a

$$D = \{\texttt{abcc}, \texttt{accb}, \texttt{baca}, \texttt{caac}, \texttt{cbcc}\}$$

Subset of strings having the same length p+1

F				L
#	a	b	c	c
#	a	c	c	b
#	b	a	c	a
#	c	a	a	c
#	c	b	c	c
a	#	b	a	c
a	a	c	#	c
a	b	c	c	#
a	c	#	c	a
a	c	a	#	b
a	c	c	b	#
b	#	a	c	c
b	a	c	a	#
b	c	c	#	a
b	c	c	#	c
c	#	a	b	c
c	#	c	a	a
c	#	c	b	c
c	a	#	b	a
c	a	a	c	#
c	b	#	a	c
c	b	c	c	#
c	c	#	a	b
c	c	#	c	b
c	c	b	#	a

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Subset of strings having the same length p+1

It preserves nice properties of Burrows-Wheeler Transform

F				L
#	a	b	c	c
a	c	c	b	b
b	a	c	a	a
c	a	a	c	c
#	a	b	c	c
a	#	b	a	c
a	a	c	#	c
a	b	c	c	#
a	c	#	c	a
a	c	a	#	b
a	c	c	b	#
b	#	a	c	c
b	a	c	a	#
b	c	c	#	a
b	c	c	#	c
c	#	a	b	c
c	#	c	a	a
c	#	c	b	c
c	a	#	b	a
c	a	a	c	#
c	b	#	a	c
c	b	c	c	#
c	c	#	a	b
c	c	#	c	b
c	c	b	#	a

$$D = \{\texttt{abcc}, \texttt{accb}, \texttt{baca}, \texttt{caac}, \texttt{cbcc}\}$$

Given P[1,p],
identify the rows prefixed by a cyclic rotation of P on the subset with string of len p+1
i.e., P[i,p]#P[1,i-1]

Subset of strings having the same length p+1

F				L
#	a	b	c	c
#	a	c	c	b
#	b	a	c	a
#	c	a	a	c
#	c	b	c	c
a	#	b	a	c
a	a	c	#	c
a	b	c	c	#
a	c	#	c	a
a	c	a	#	b
a	c	c	b	#
b	#	a	c	c
b	a	c	a	#
b	c	c	#	a
b	c	c	#	c
c	#	a	b	c
c	#	c	a	a
c	#	c	b	c
c	a	#	b	a
c	a	a	c	#
c	b	#	a	c
c	b	c	c	#
c	c	#	a	b
c	c	#	c	b
c	c	b	#	a

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Strings corresponding to these rows are at distance one from P (an insertion)!

Subset of strings having the same length p+1

F				L
#	a	b	c	c
#	a	c	c	b
#	b	a	c	a
#	c	a	a	c
#	c	b	c	c
a	#	b	a	c
a	a	c	#	c
a	b	c	c	#
a	c	#	c	a
a	c	a	#	b
a	c	c	b	#
b	#	a	c	c
b	a	c	a	#
b	c	c	#	a
b	c	c	#	c
c	#	a	b	c
c	#	c	a	a
c	#	c	b	c
c	a	#	b	a
c	a	a	c	#
c	b	#	a	c
c	b	c	c	#
c	c	#	a	b
c	c	#	c	b
c	c	b	#	a

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Subset of strings having the same length p+1

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Space: $nH_k + n \cdot o(\log \sigma)$

Time: $O(|P| \log \log \sigma + occ)$

Open Problems

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Solution based on Patricia tries and Perfect

Time $O(|P| + occ)$

Space $2nH_k + n \cdot o(\log \sigma) + 2d \log d$

Top-k strings in $O(|P| + k \log k)$

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Open Problems

Optimal time/space complexities?

Time: $O(|P| \log \sigma / w + occ)$ where w size of a word

Space: $O(n \log \sigma)$

Thank You