Pseudo-realtime Pattern Matching: Closing the Gap

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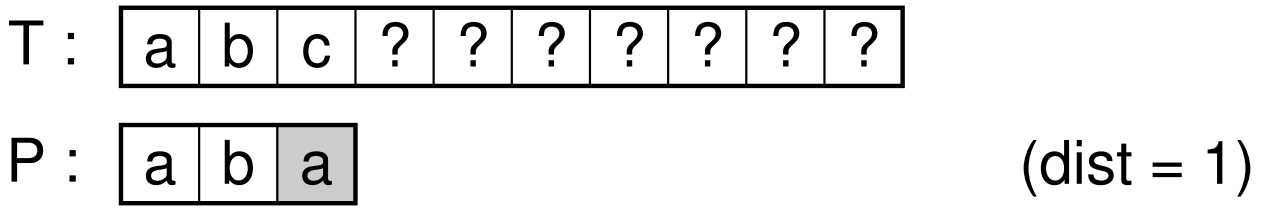
University of Bristol

Introduction: Pseudo-realtime pattern matching

- Consider a text, T (length n) and a pattern P (length m)
- We assume we have P in advance but T arrives online...



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(Hamming distance shown)

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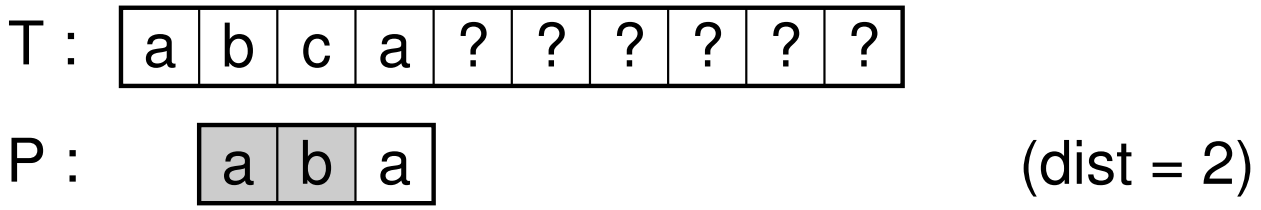

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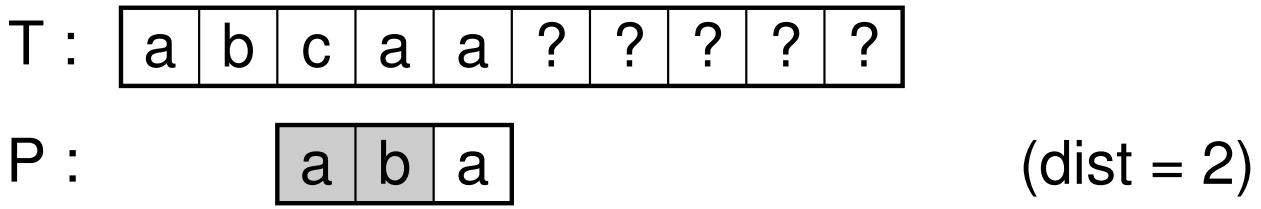

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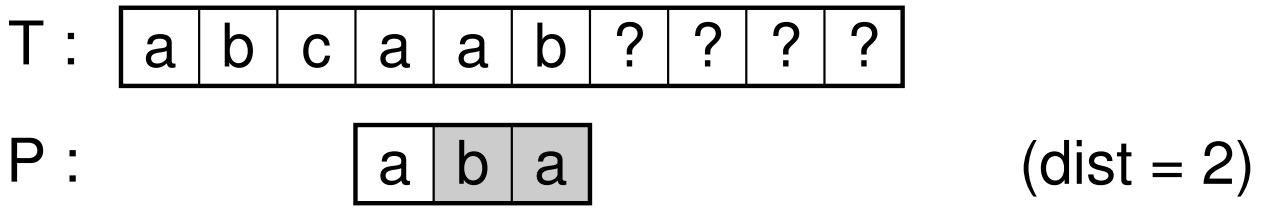

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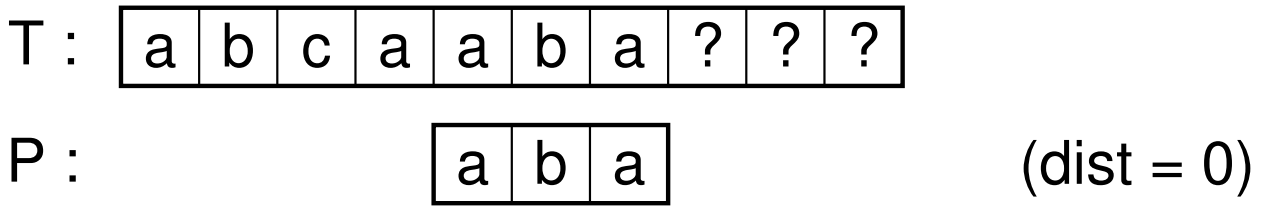

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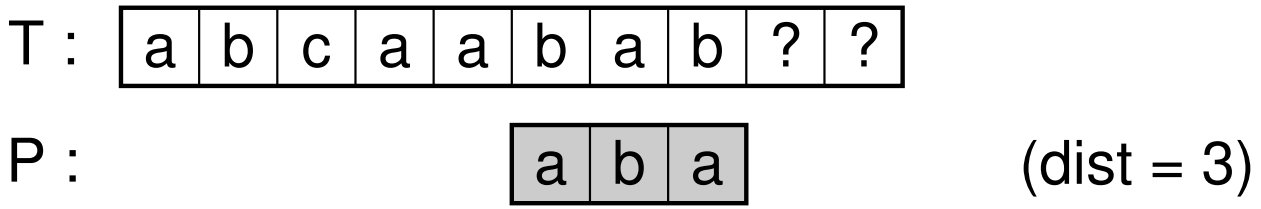

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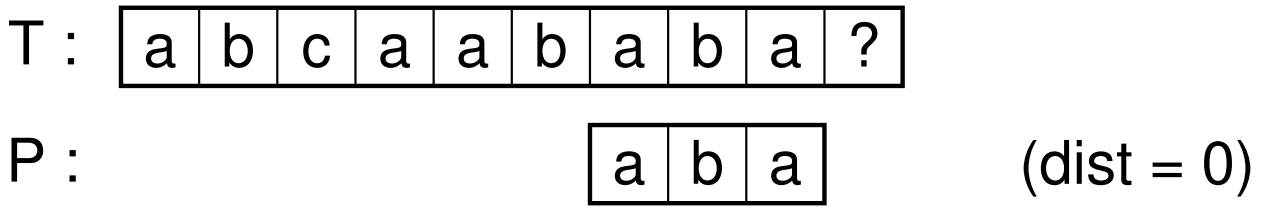

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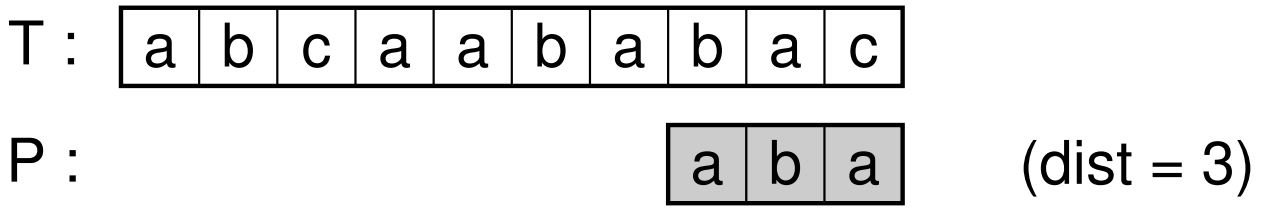

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Previous and Related work

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Local, Deterministic (C., Efremenko, Porat and Porat. CPM 2008)			
Matching with wildcards	$O(\log m)$	$O(\log m)$	$O(m)$
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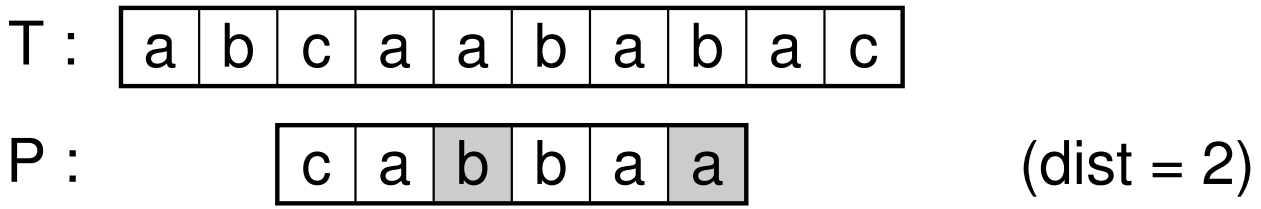
Can we improve the k-mismatch result?

- The k-mismatch problem is to find all alignments of P with T where the **Hamming distance** is at most k.



- Offline: $O(n\sqrt{k \log k})$ time[1].
- Pseudo-realtime: $O(\sqrt{k \log k} \log m)$ time per character.

Problem: k is often very small in comparison with m.

[1] Amir, Lewenstein, Porat. SODA 2000

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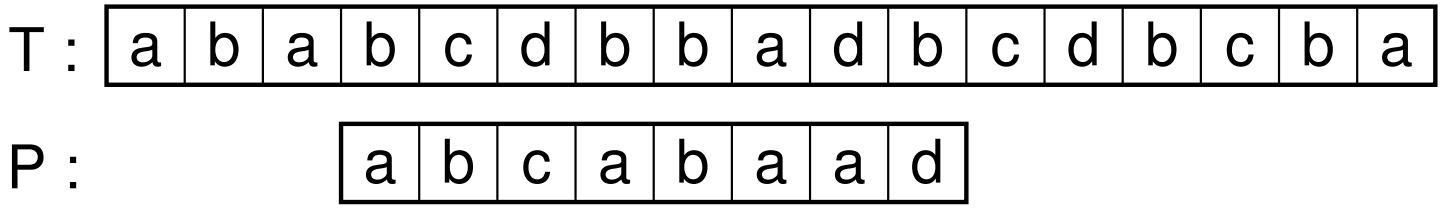

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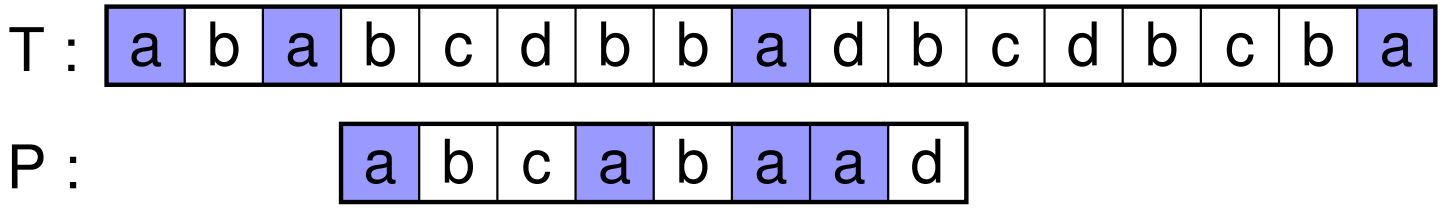

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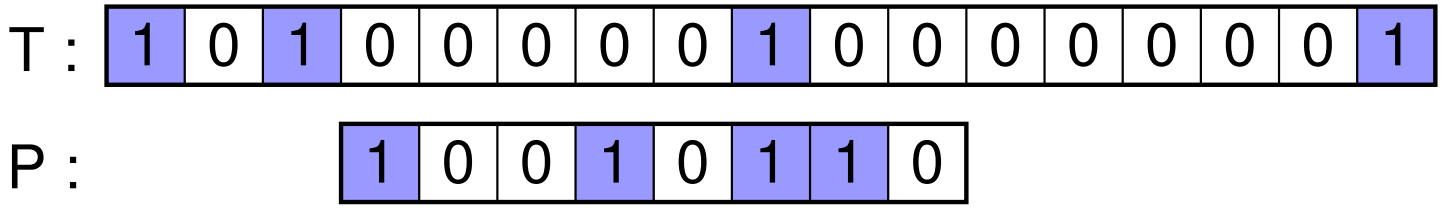

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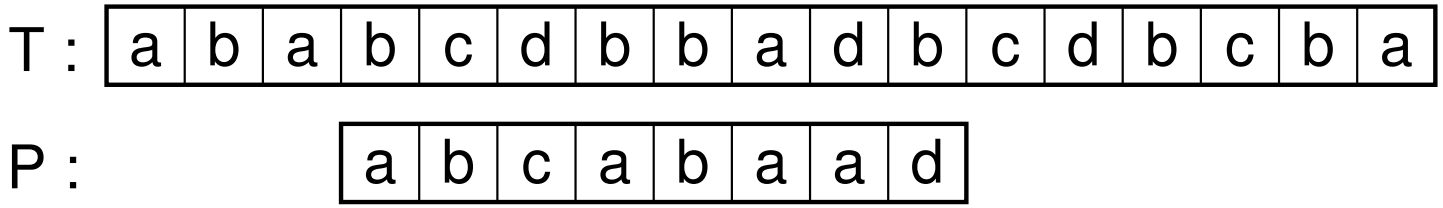

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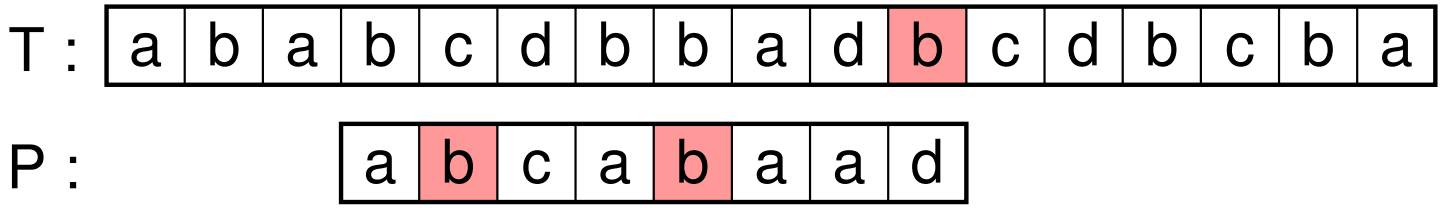

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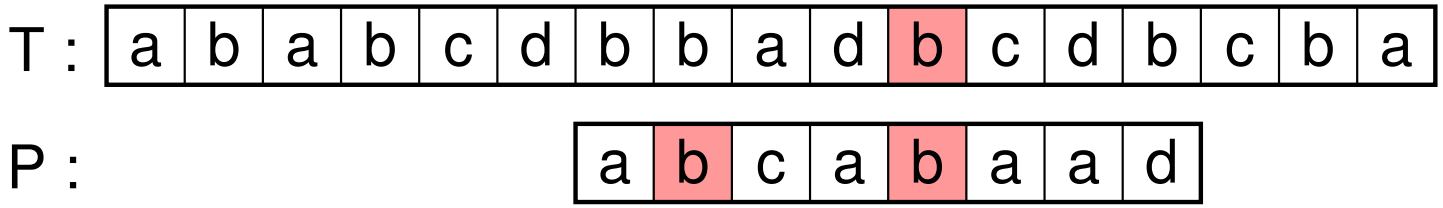

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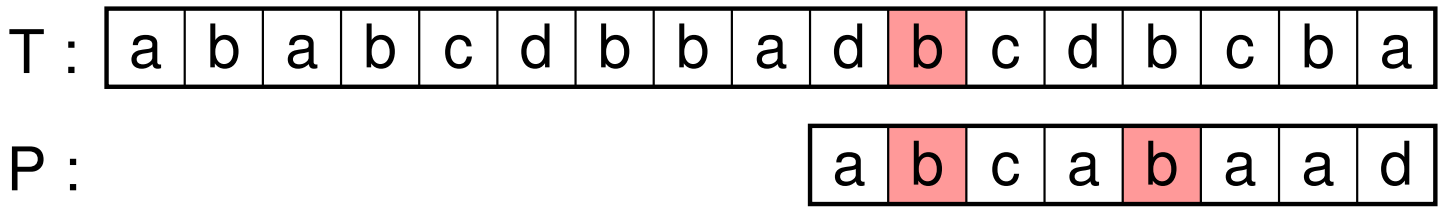

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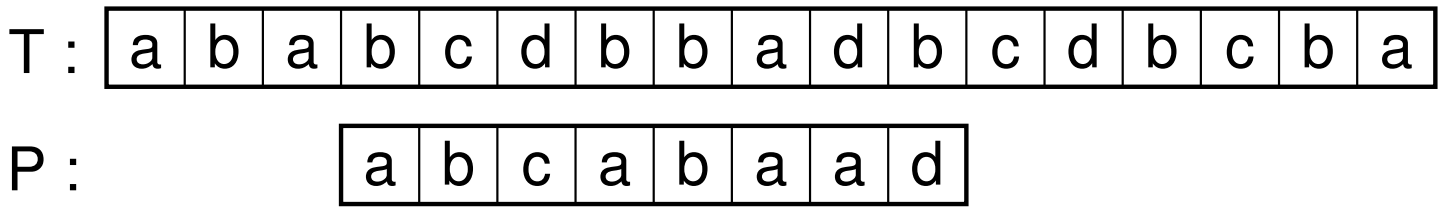

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This method gives a time complexity of $O(n\sqrt{k} \log m + n\sqrt{k})$

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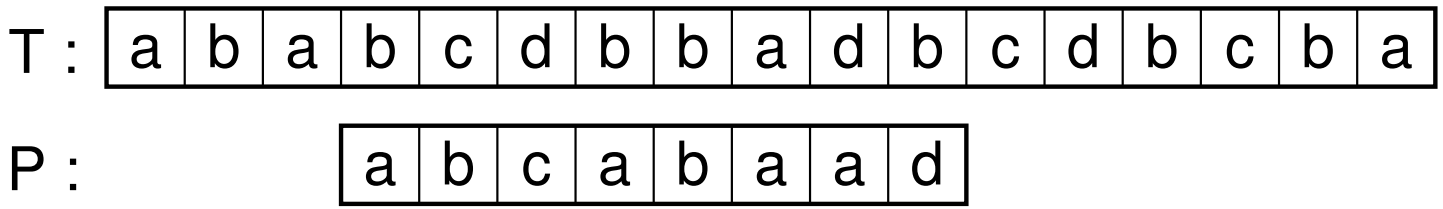

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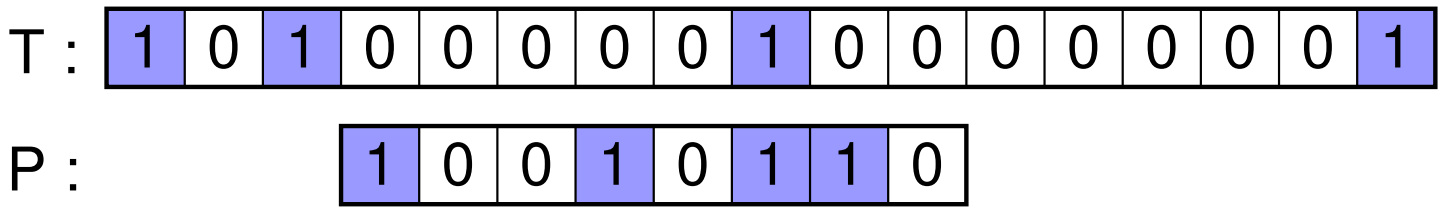
How do we do cross-correlations *online*?

The cross-correlation between arrays T and P is defined by:

$$(T \otimes P)[i] = \sum_{j=0}^{m-1} T[i+j]P[j]$$



- Offline: $O(n \log m)$ total time (via FFTs).
- Pseudo-realtime: $O(\log^2 m)$ time per character[2].

[2] via the blackbox method of CEPP 2008

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T :	a	b	a	b	c	d	b	b	a	d	b	c	d	b	c	b	a

P :			a	b	c	a	b	a	a	d

- Matches with a single *frequent* character can be found using **PsR cross-correlations** in $O(\log^2 m)$ **time per character**.
- Each *infrequent* text character can't match very many times.

This gives a time complexity of $O(\sqrt{k} \log m + \log^2 m)$ per character.

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- Check each location using $(k+1)$ *Longest Common Extension* queries. Offline we can preprocess T, P for $O(1)$ time queries.

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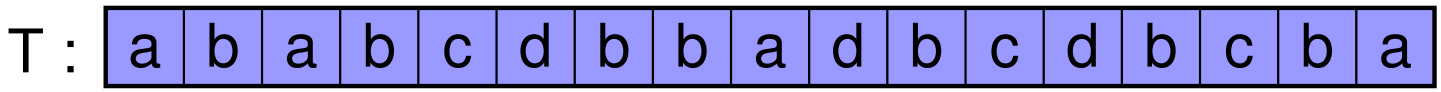

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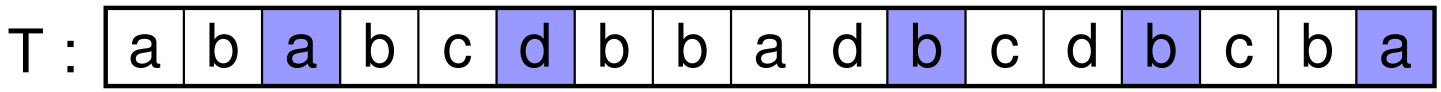

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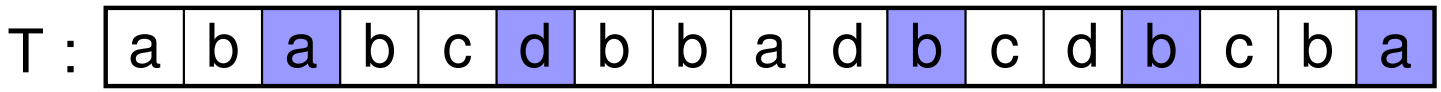

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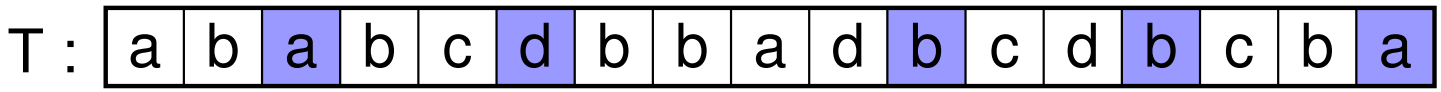

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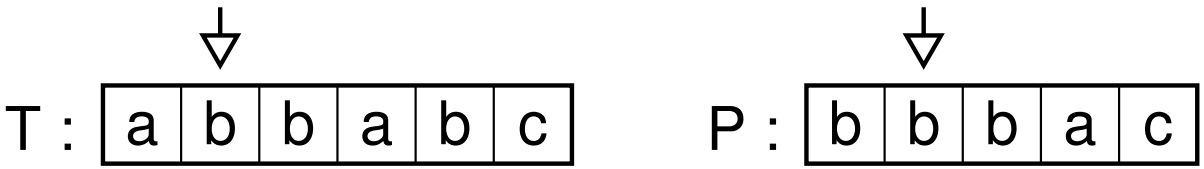

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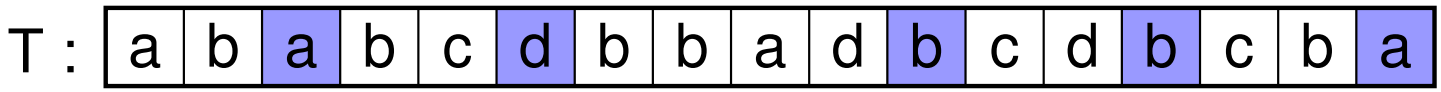

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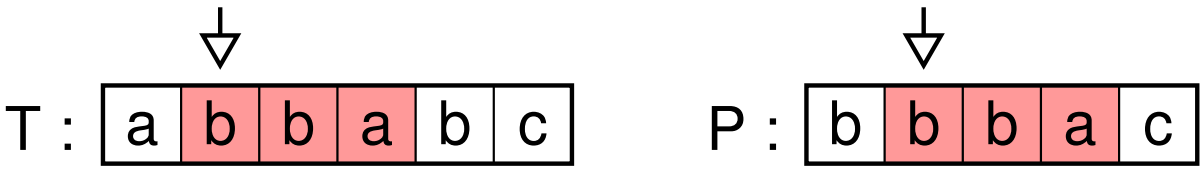

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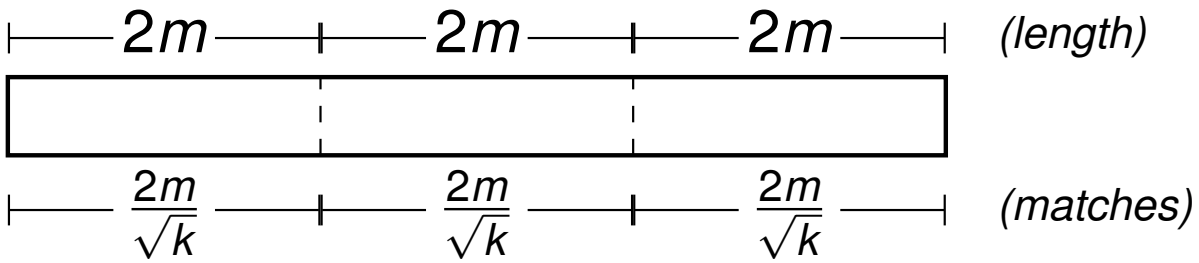

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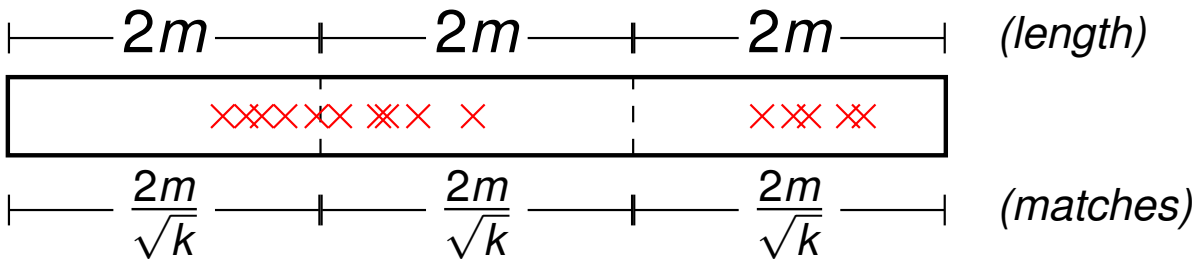

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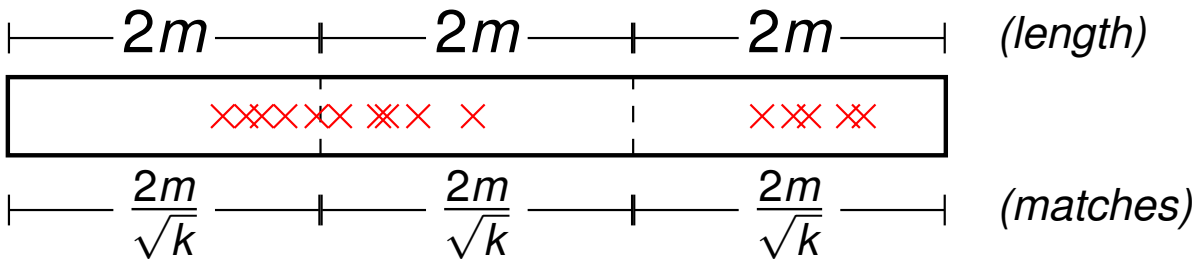

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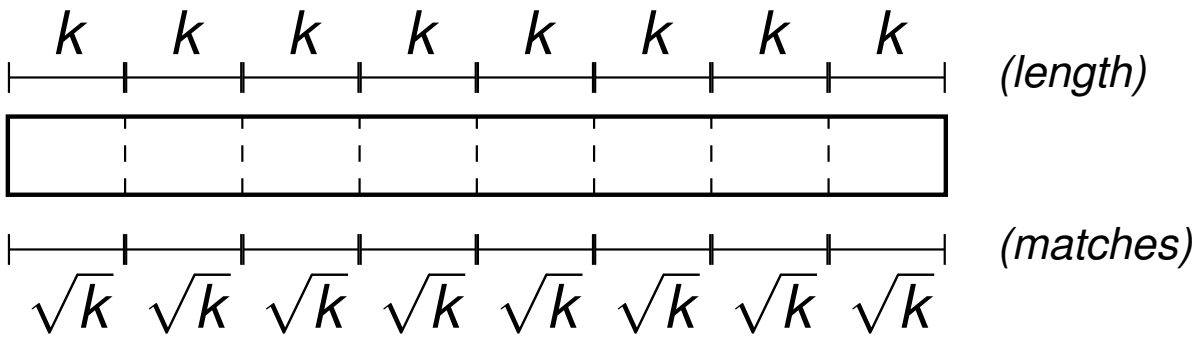

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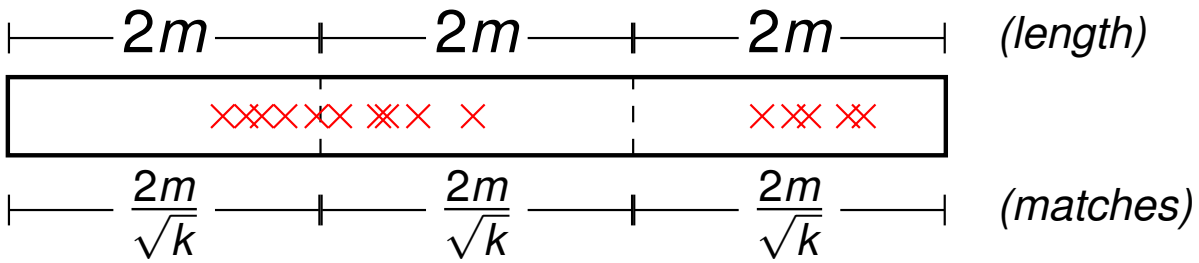

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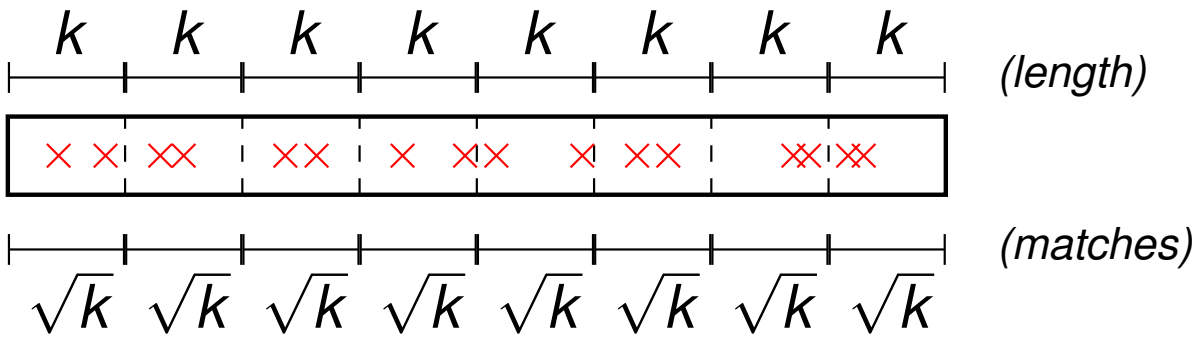

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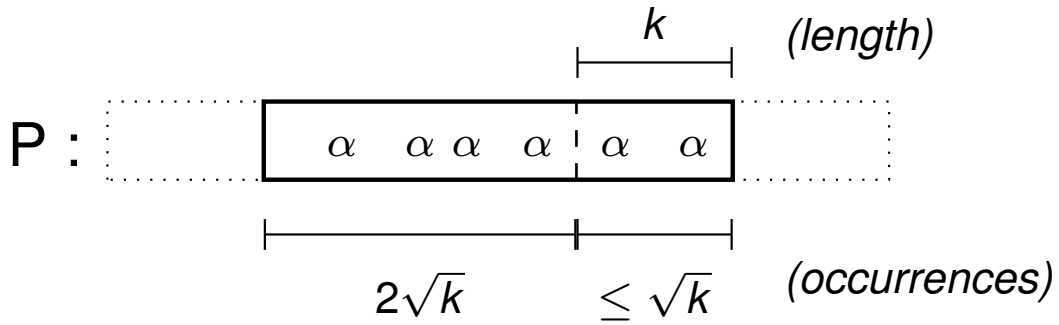

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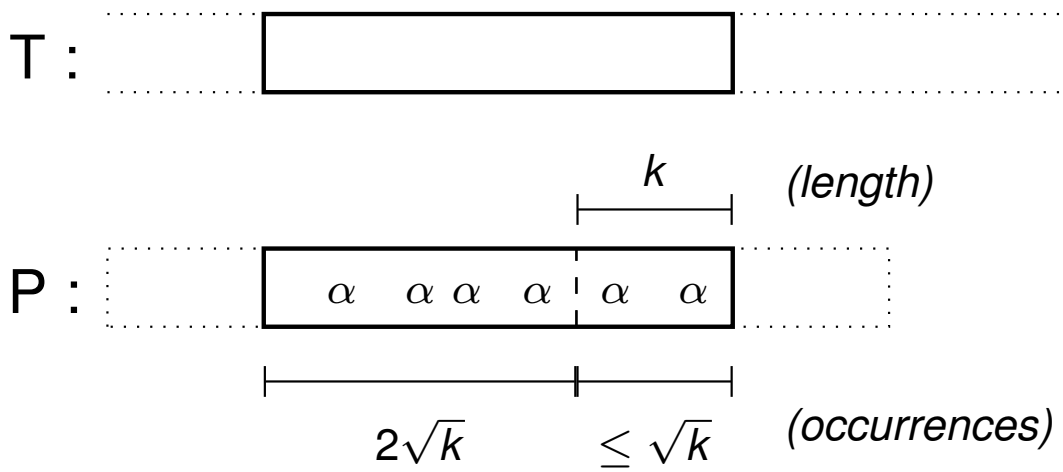

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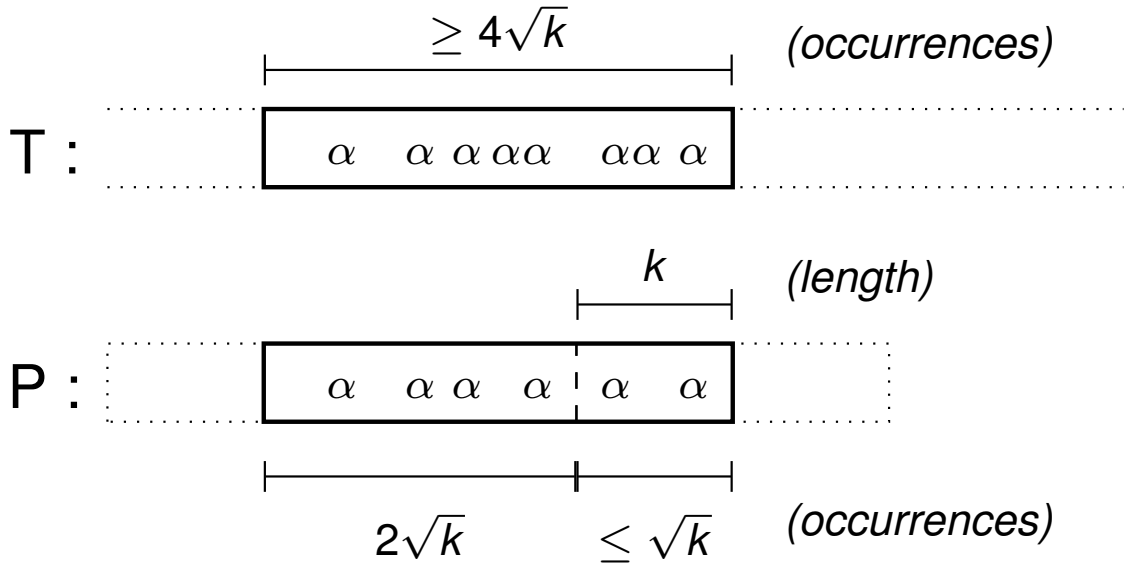

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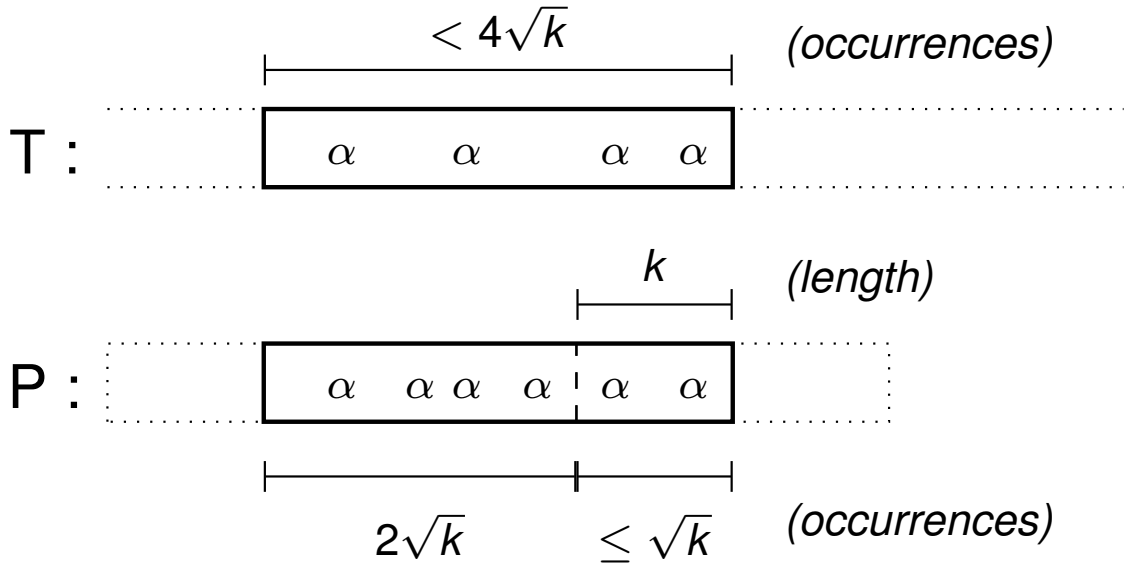

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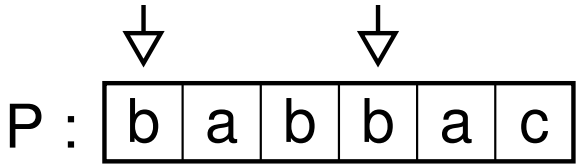

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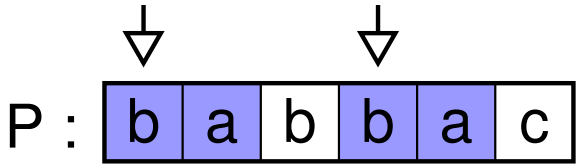

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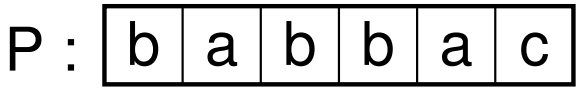

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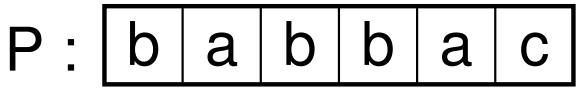

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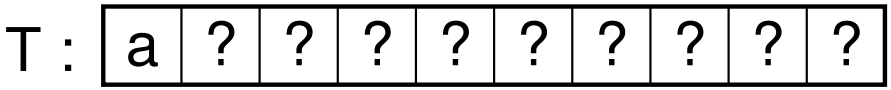

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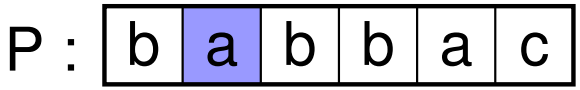

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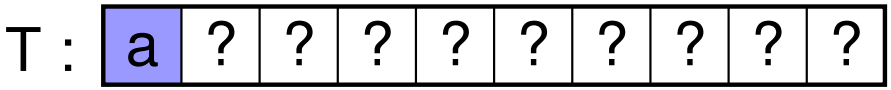

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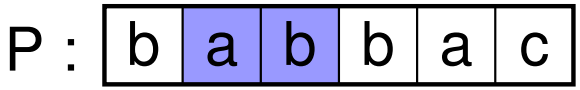

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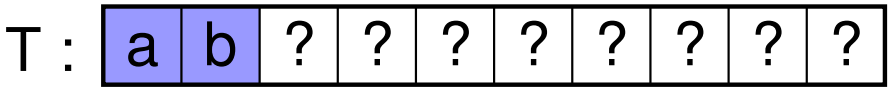

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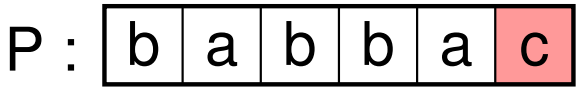

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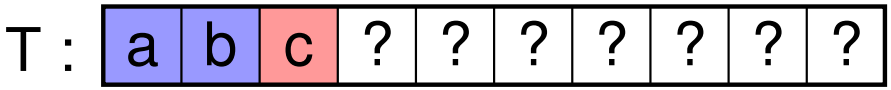

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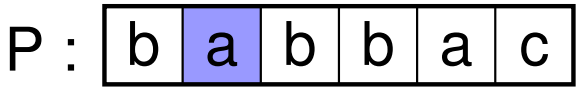

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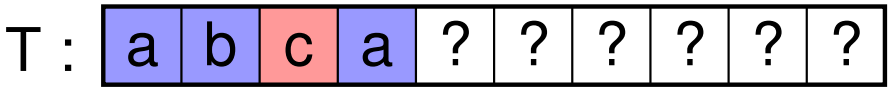

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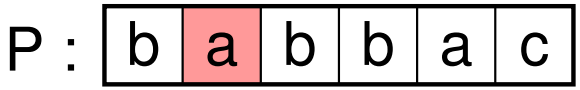

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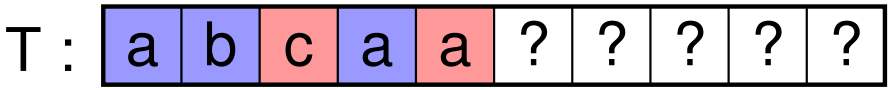

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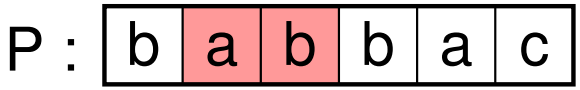

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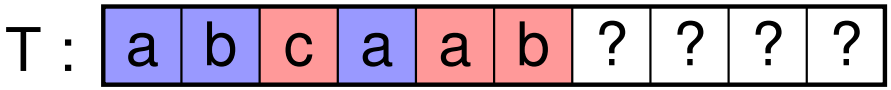

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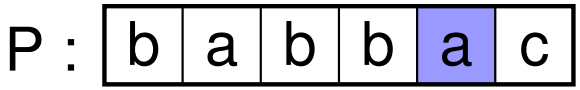

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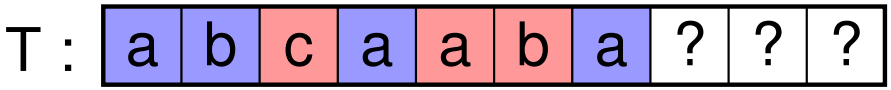

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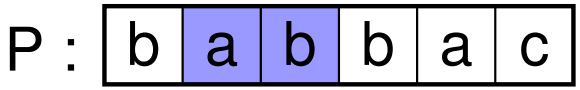

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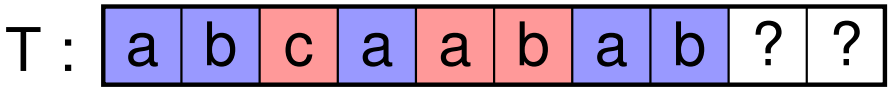

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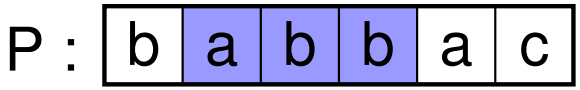

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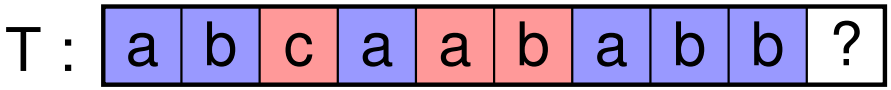

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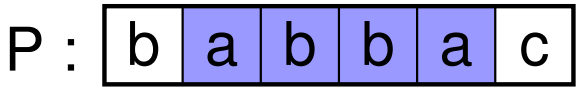

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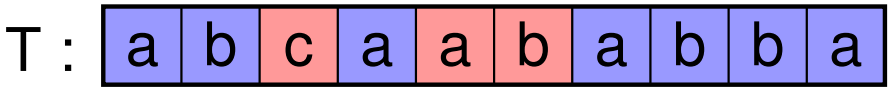

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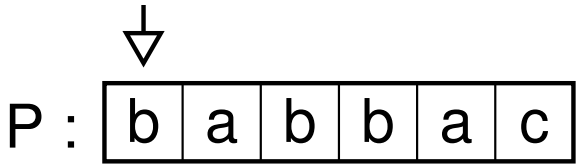

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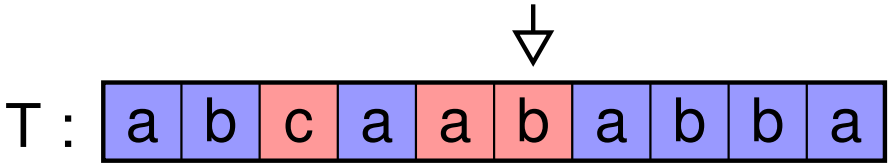
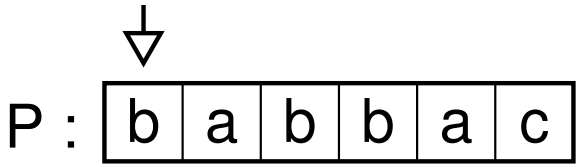


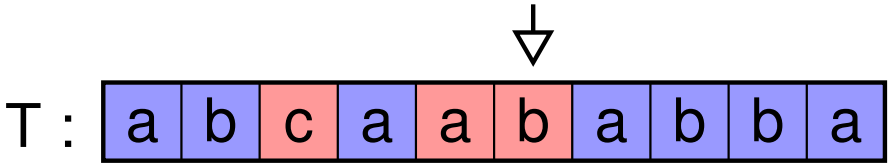
- How many self queries do we need to do?

What is wrong with the original *LCE* queries?

Offline, complex text preprocessing gives constant *LCE* query times. Instead we preprocess the pattern for *self-LCE* queries. . . .



Main Idea: Encode the arriving text in terms of the pattern and use *self-LCE* queries on the pattern to perform pattern/text *LCE* queries.



- How many self queries do we need to do? *At most three!*

Putting it all together

The remainder is conceptually simple but the details are *fiddly*:

Conceptually

- Use the improved filter on each k-length text substring online.
- Check the $\sqrt{k}$ potential matches using the online *LCE* queries.

However, we need to be very careful about the scheduling details to ensure that the work is done in time.

This results in a time complexity per text character of

$$O(\sqrt{k}\log m + \log^2 m)$$

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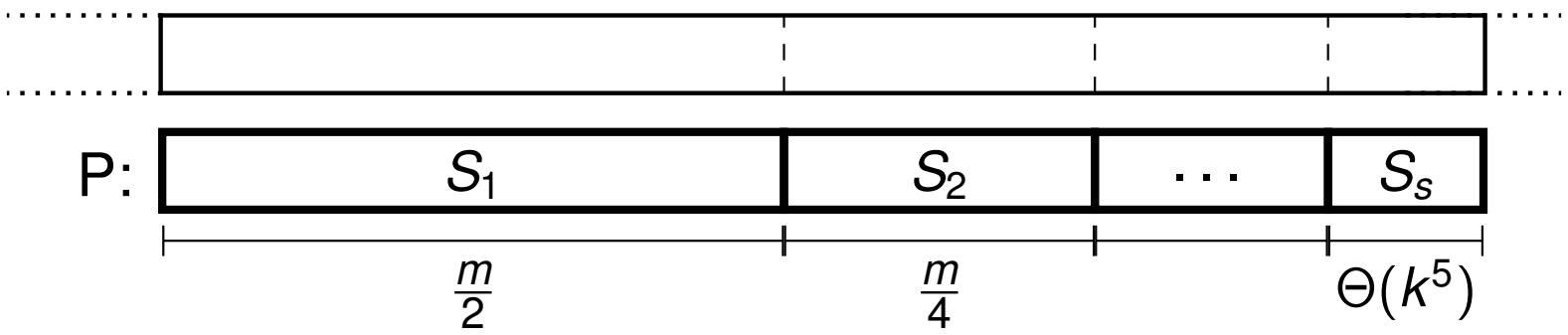
Didn't you say additive $\log m$*?*

How do we get an additive log m?

- Split the pattern into $s \in O(\log m)$ consecutive subpatterns where each subpattern is half the length of the previous.
- We set s so that $k^5/2 \leq |S_s| < k^5$.



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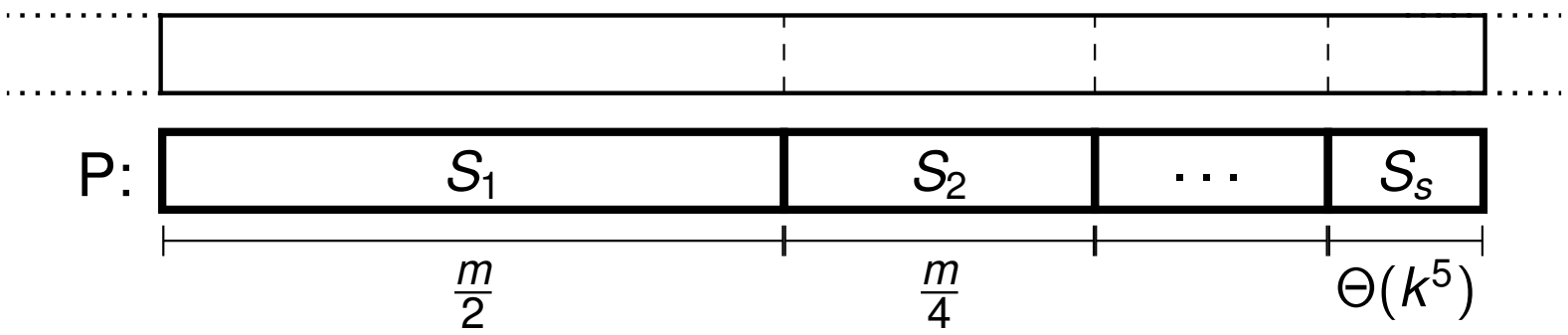

- Use the $O(\sqrt{k}\log m + \log^2 m)$ algorithm for the final subpattern.

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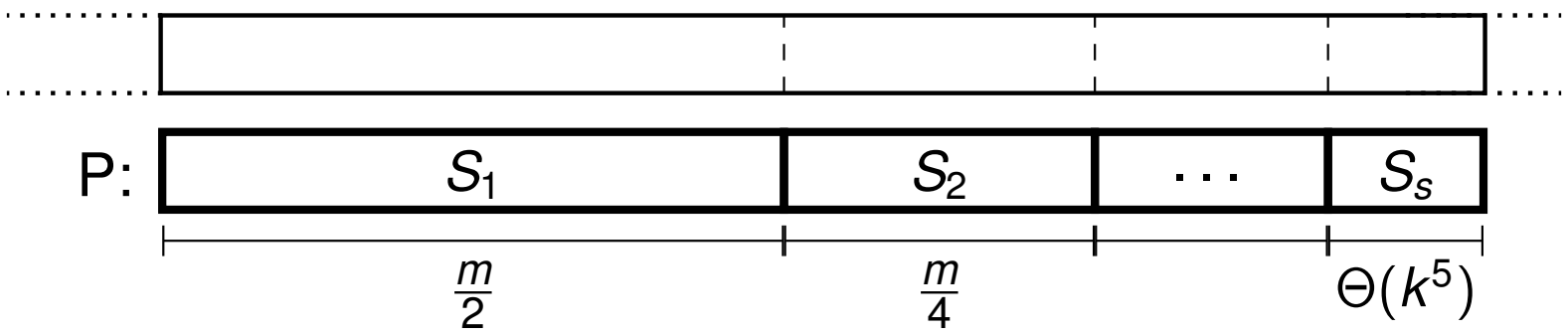
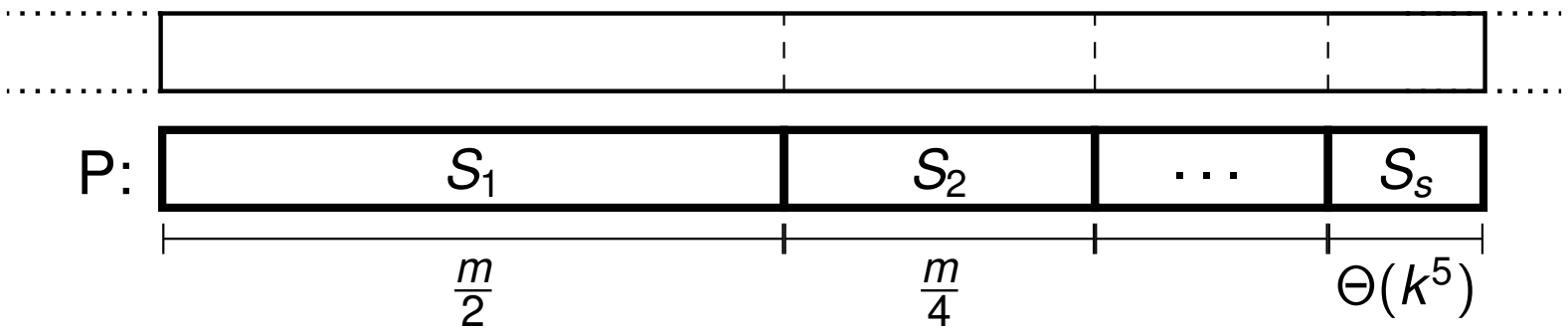


- Use the $O(\sqrt{k}\log m + \log^2 m)$ algorithm for the final subpattern.
- As $|S_s| \in \Theta(k^5)$ we have $O(\sqrt{k}\log k)$ time per character.

How do we get an additive log m?

- What about mismatches with subpatterns $S_1, S_2, \ldots, S_{s-1}$?
- Each of these subpatterns has length greater than k^5.



- For the case where $m >> k$ there is an offline algorithm[3] with time complexity $O(n + nk^4 \log k/m)$.
- Applied to one of our subpatterns (offline) this would be $O(n)$.
- Using the *black box method* we obtain $O(1)$ time per character.

[3] Amir, Lewenstein and Porat. SODA 2000

Conclusions

k-mismatch

- Summing across all subpatterns we obtain a time complexity of

$$O(\sqrt{k}\log k + \log m) \text{ per character (and } O(m) \text{ space).}$$

- This is tantalisingly close to the $O(n\sqrt{k\log k})$ offline solution.

k-differences

- By combining our realtime *LCE* processing with an incremental k-difference algorithm[4] we obtain a time complexity of

$$O(k) \text{ time per character (and } O(m) \text{ space).}$$

- This is matches the $O(nk)$ offline solution.

[4] Landau, Myers and Schmidt, SIAM J. Comput 1998