Fast Searching in Packed Strings

Philip Bille

String Matching

- Problem: Given strings P and Q of lengths m and n, resp., report all occurrences of P in Q.

$$Q = \mathtt{aababcabbababcababcaaabbab}$$

$$P = \mathtt{ababca}$$

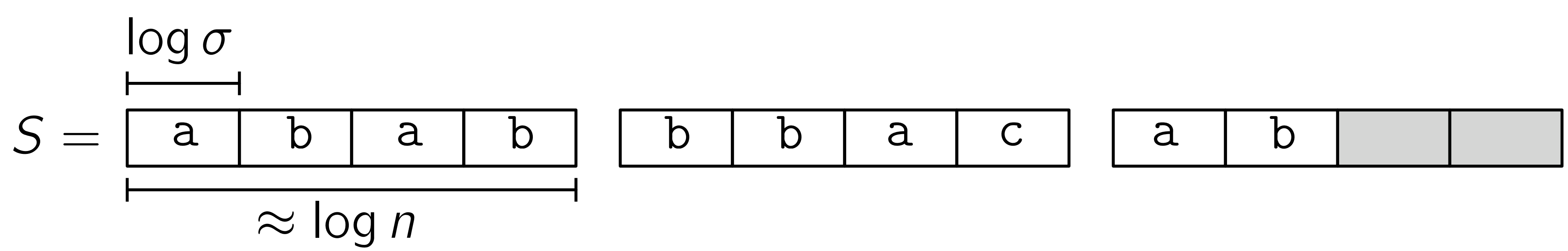
- KMP-algorithm [KMP1977] uses O(n) time (assume w.l.o.g. m ≤ n).

- Optimal if strings are stored with one char per memory word.

Packed Strings

- Real strings are *packed*:

$S = \texttt{ababbbacab}$



- With word-length log n a memory word holds ≈ log n / log σ characters.

- S uses $O(|S| \log \sigma / \log n) = O(|S| / \log_\sigma n)$ words.

Packed String Matching

- Problem: String matching with P and Q in packed representation.
- Lower bound: $\Omega\left(\frac{n+m}{\log_\sigma n} + \mathrm{occ}\right)$
- What is the best upper bound?
- Can we do better than O(n)?

A Simple Algorithm: Use a lots of space

$P =$ `ababca`

$Q =$

a	b	a	b	b	b	a	c	a	b		
			4				0		2		

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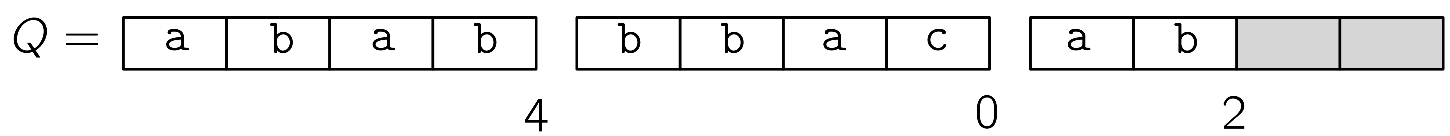

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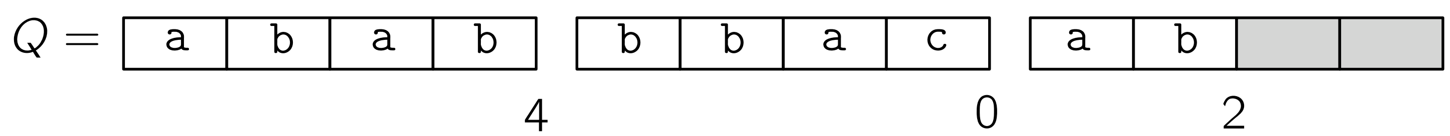

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- To do step in constant time store for each prefix of P and each combination of r characters a pointer to the next prefix. (called a "super-alphabet technique" [Fre02]).

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Time: $O(n/r + m\sigma^r + \mathrm{occ})$

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		$r = \epsilon \log_\sigma n$
Space:	$O(m\sigma^r)$	$O(mn^\epsilon)$
Time:	$O(n/r + m\sigma^r + \text{occ})$	$O(n/\log_\sigma n + mn^\epsilon + \text{occ})$

Complexities

Time	Space	
$O\left(\frac{n}{r} + m\sigma^r + \mathrm{occ}\right)$	$O(m\sigma^r)$	Simple
$O\left(\frac{n}{\log_\sigma n} + mn^\varepsilon + \mathrm{occ}\right)$	$O(mn^\varepsilon)$	
$O\left(\frac{n}{r} + m + \sigma^r + \mathrm{occ}\right)$	$O(m + \sigma^r)$	This paper
$O\left(\frac{n}{\log_\sigma n} + m + \mathrm{occ}\right)$	$O(m + n^\varepsilon)$	

Algorithm Overview

- Based on the Knuth-Morris-Pratt automaton.

- The “Four-Russian Technique” (divide and tabulate) with new twists.

The Knuth-Morris-Pratt Automaton

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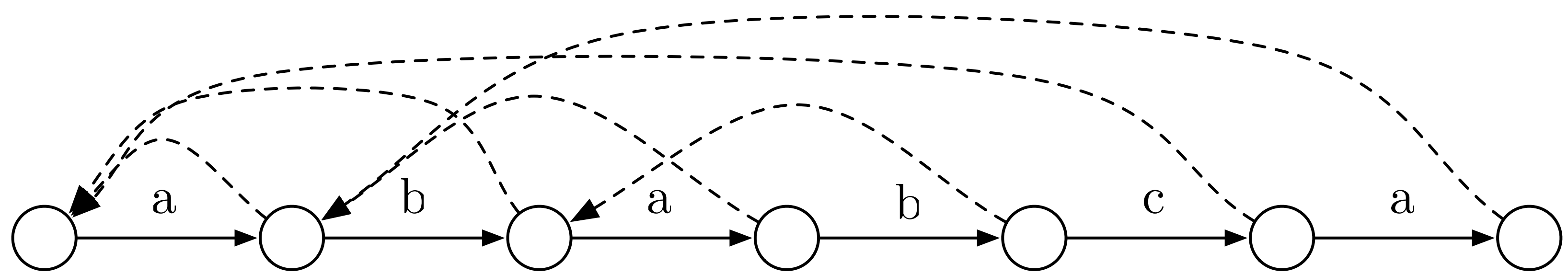

KMP(P)

A First Attempt: The Four-Russian Technique

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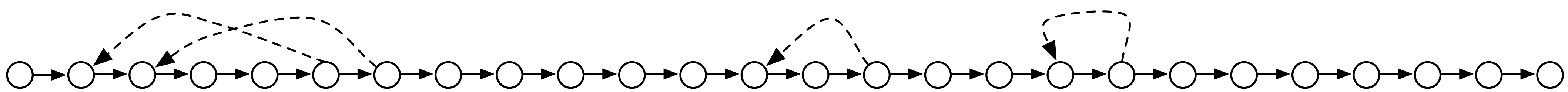

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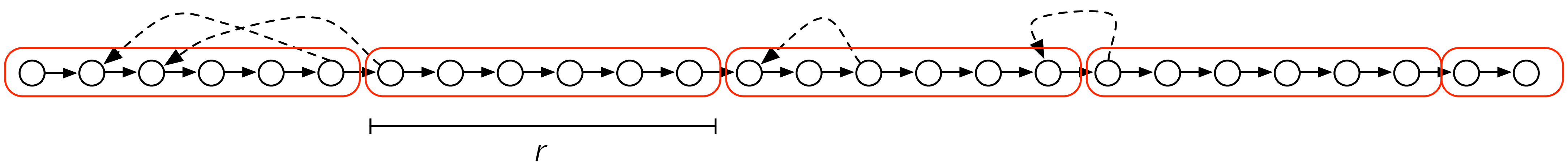

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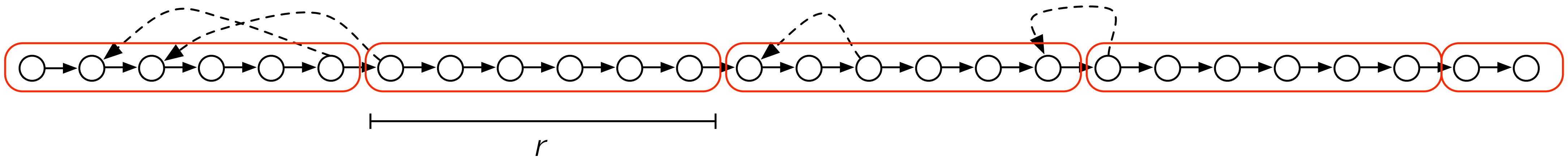

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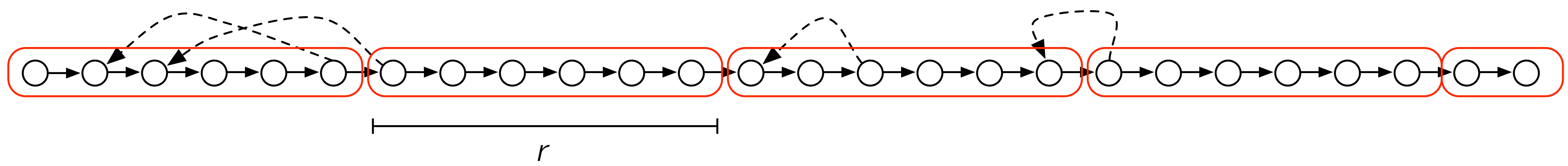

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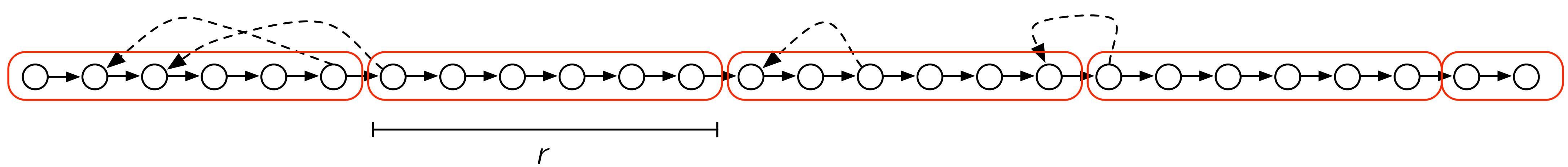

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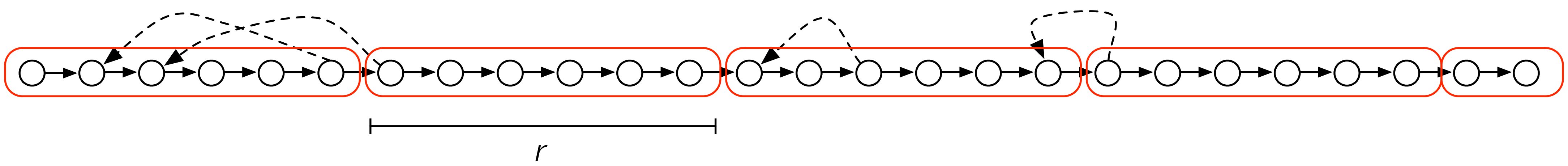

- Tabulate information for each subautomata to allow up to r *internal* transitions in constant time.

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- Issue 1: Too many external transitions.

- Issue 2: Representing subautomata compactly.

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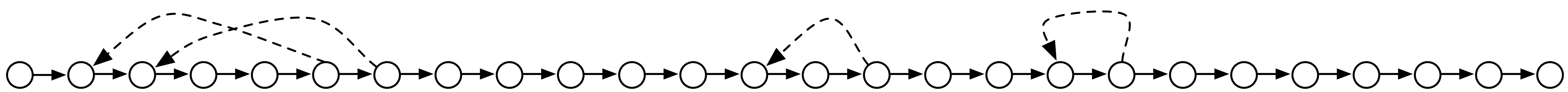

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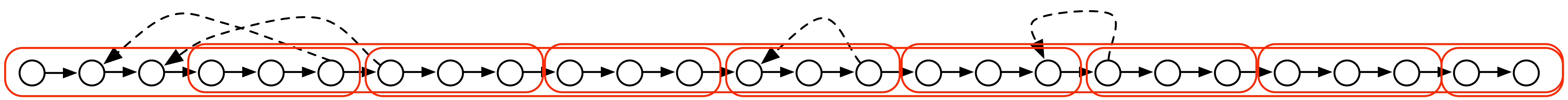

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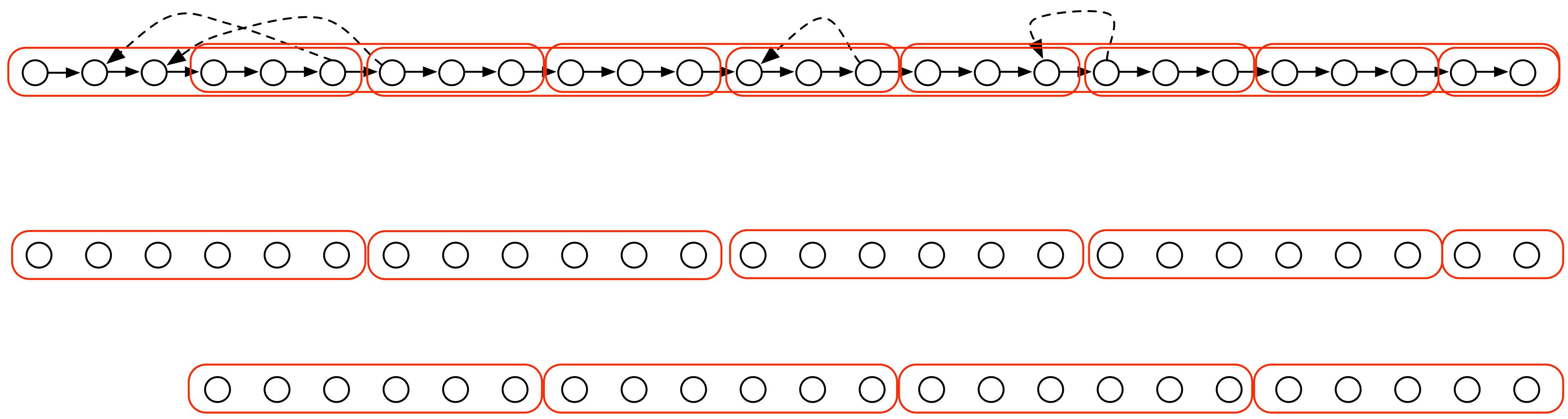

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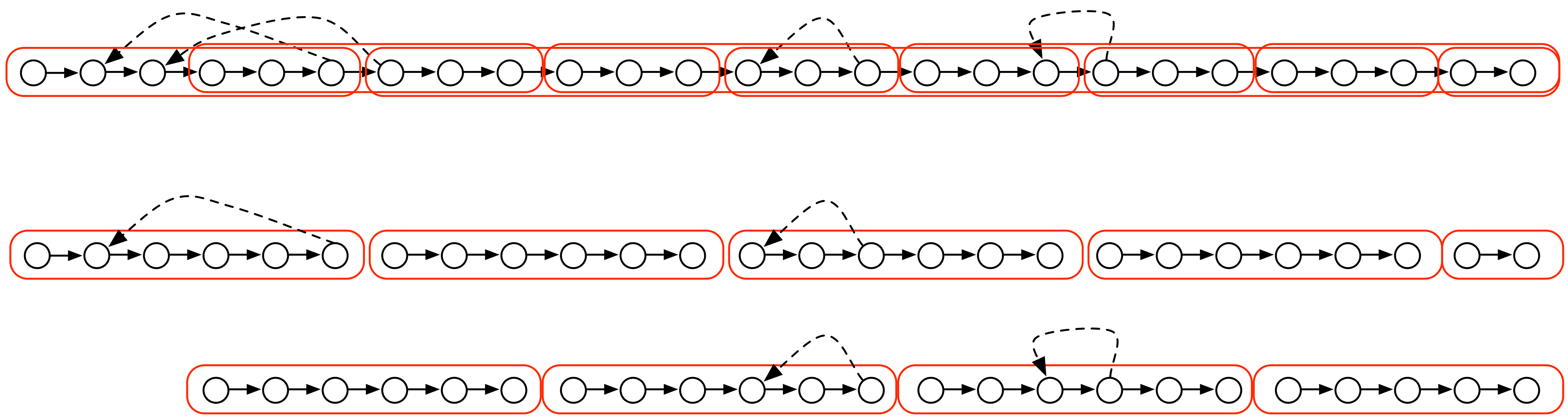

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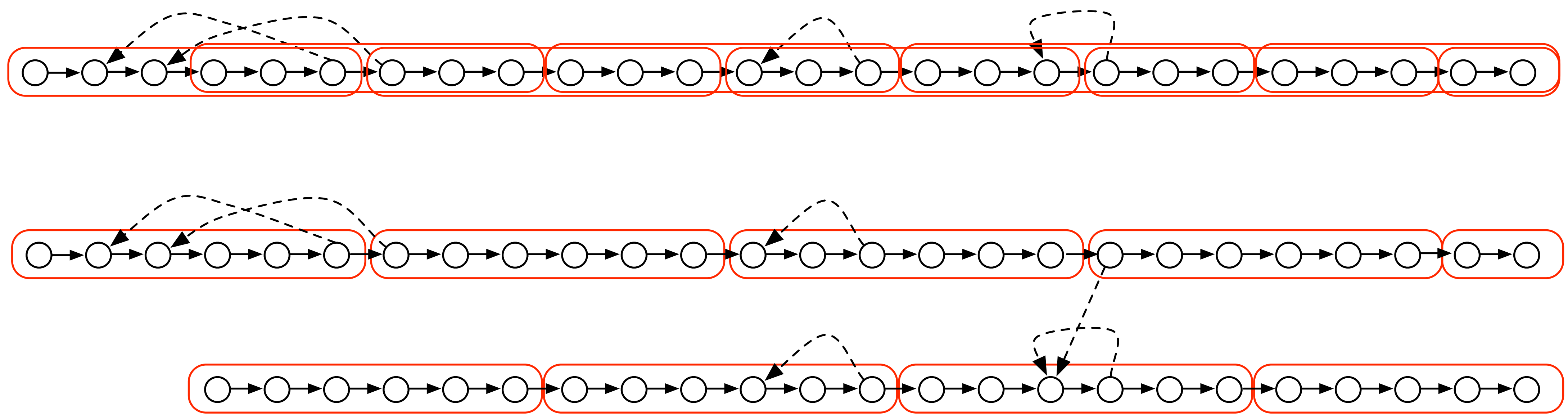

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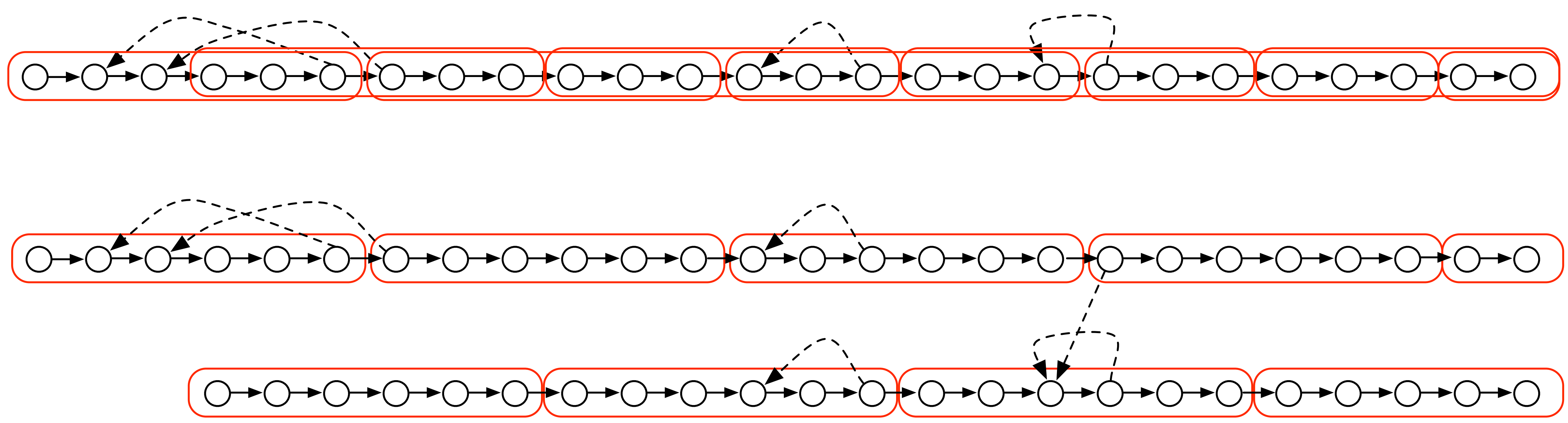
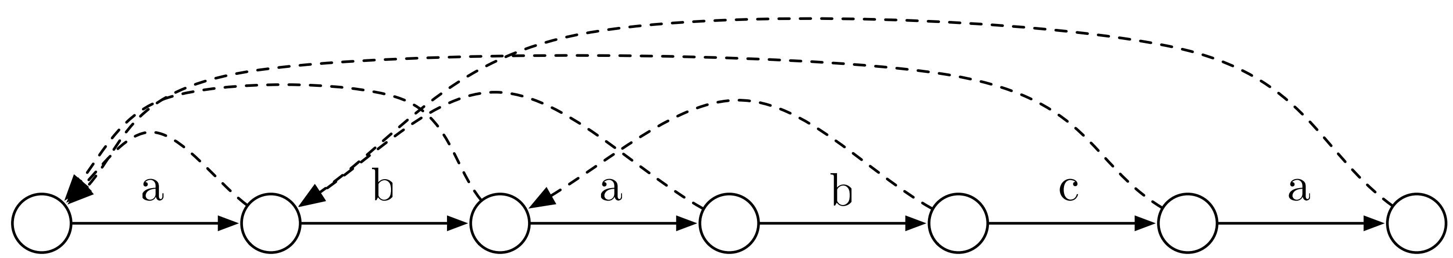


At most O(n/r) external transitions in simulation of Q

Fixing 2: Representing Subautomata Compactly



- We want to encode an arbitrary subautomaton of KMP(P) in O(r log σ) bits.
- Non-failure transitions encoded by the sequence of labels in O(r log σ) bits.
- How about the failure transitions in S?

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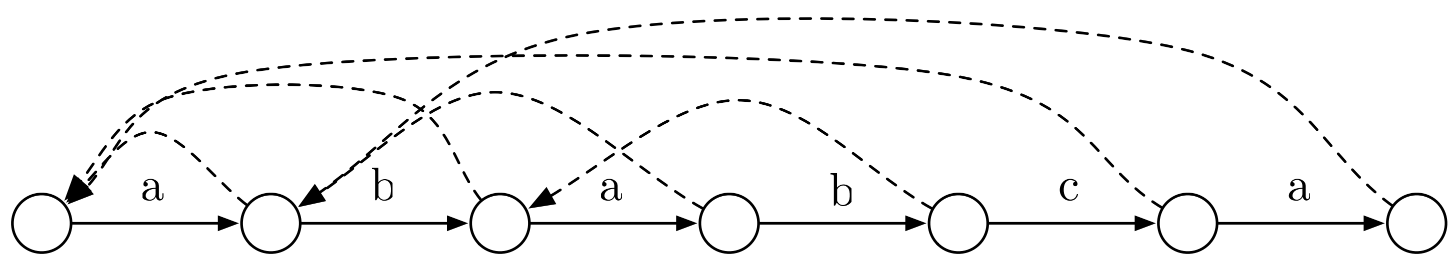

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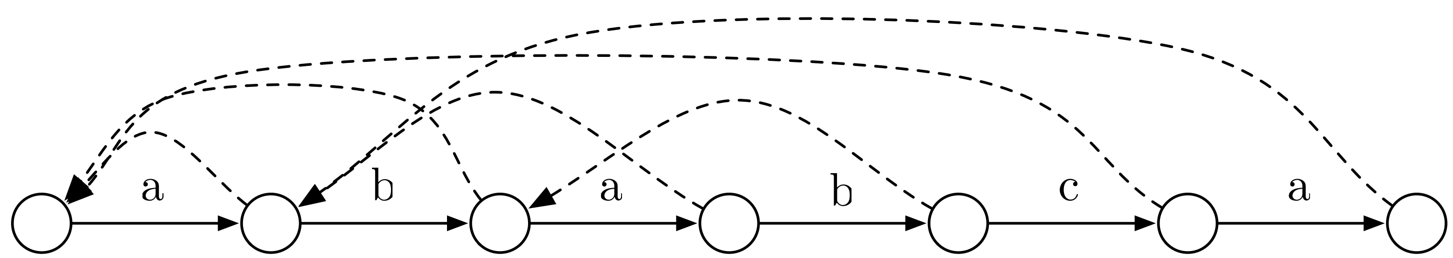

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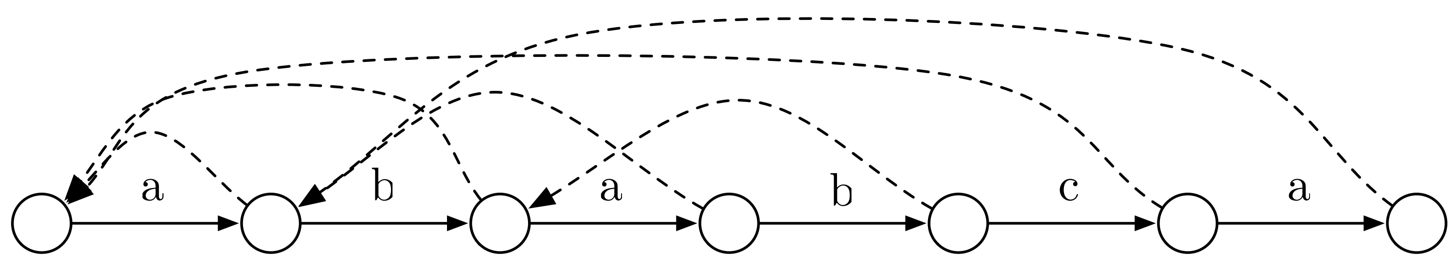

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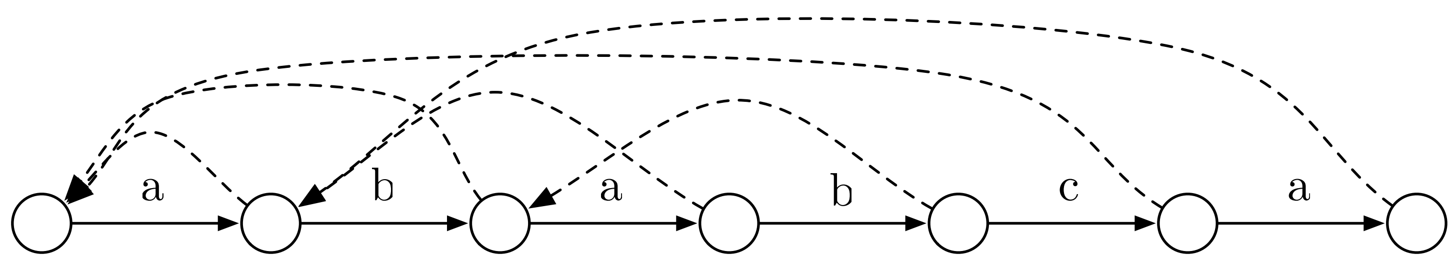

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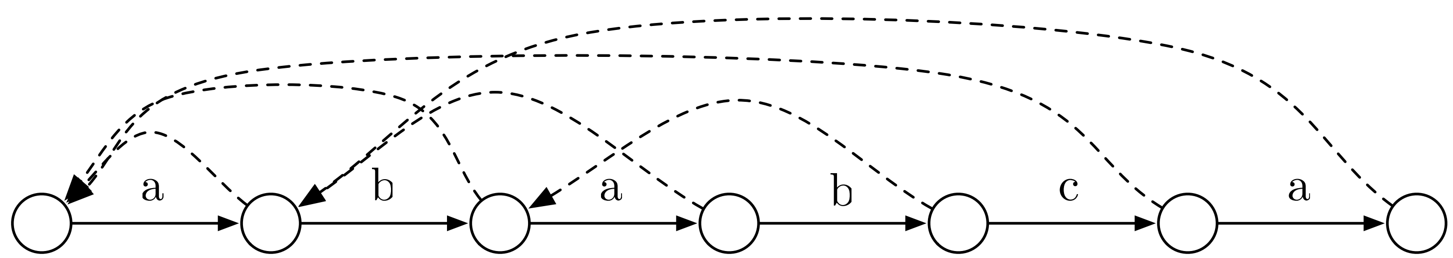

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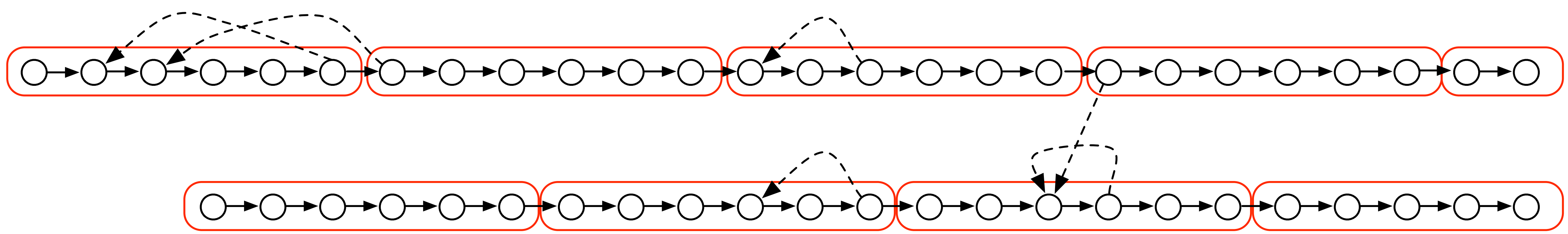
- Instead we exploit a basic property of KMP-automata: In any subautomaton failure transition endpoints increase by at most 1 between consecutive states.
- => Total increase at most r => Total decrease at most O(r).
- => We can difference encode all failure transitions with O(r) bits.

Putting the Pieces together



- Construct segment automaton and tabulate transitions for subautomata using the compact encoding.

- Simulate the segment automaton. Each external transitions is done explicitly. Internal transitions are done using the tabulation.

- Complexity:

$$r = \epsilon \log_\sigma n$$

Space:	$O(m + \sigma^r)$	$O(m + n^\epsilon)$
Time:	$O(n/r + m + \sigma^r + \text{occ})$	$O(n/\log_\sigma n + m + \text{occ})$

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- Packed problems appear everywhere.
  - Longer word lengths => more packing.
  - Most packed problems are not well-solved.