Dynamic Entropy-Compressed Sequences and Full-Text Indexes

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Outline

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Background

Succinct Data Structures

- Compacted to take little space.
- Must retain their functionality and direct accessibility.
- Improve performance considering memory hierarchy.
- Increasingly popular for technological reasons.
- Succinct $\Rightarrow$ little space on top of the raw data.
- Compressed $\Rightarrow$ space proportional to the data entropy.

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Binary Sequences

- Let $B = b_1 b_2 \ldots b_n$ be a binary sequence.
- $rank_b(B, i) =$ number of occurrences of b in $B[1, i]$.
- $select_b(B, j) =$ position of j-th occurrence of b in B.
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- Raman, Raman and Rao, SODA 2002.
- Divides the sequence into blocks...
- ... and uses variable-length representation for them.
- Take $nH_0 + o(n)$ bits of space.
- Solve *rank* and *select* in constant time.
- Static solution.

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- A balanced tree with leaves managing $O(\log n)$ bits.
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Main Result

Our contribution

- A structure structure that takes $nH_0 + o(n)$ bits of space.
- It performs *rank*, *select*, *insert* and *delete* in $O(\log n)$ time.
- The first nH_0-size dynamic data structure for binary sequences.
- Many applications, details soon.

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Main line of work

Similar to Chan, Hon, and Lam, but...

- Leaves must hold more than $O(\log n)$ bits, so the overhead of tree pointers can be $o(n)$.
- Then leaves cannot be processed bitwise in $O(\log n)$ time.
- In the leaves, we cannot have space for $(1+\epsilon)S$ bits and use just S.
- Then we cannot easily handle leaf overflows in constant time.

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Data structure and queries

- Balanced binary tree.
- Leaves handle blocks of $f(n) \log n$ bits.
- Internal nodes know subtree size and weight.
- Extra space of the tree is $O(n/f(n))$ bits.
- *rank* and *select* are solved by a top-down traversal...
- ... plus a linear scan over the leaf sequence.
- 4-Russians technique permits scanning in $O(f(n))$ time.

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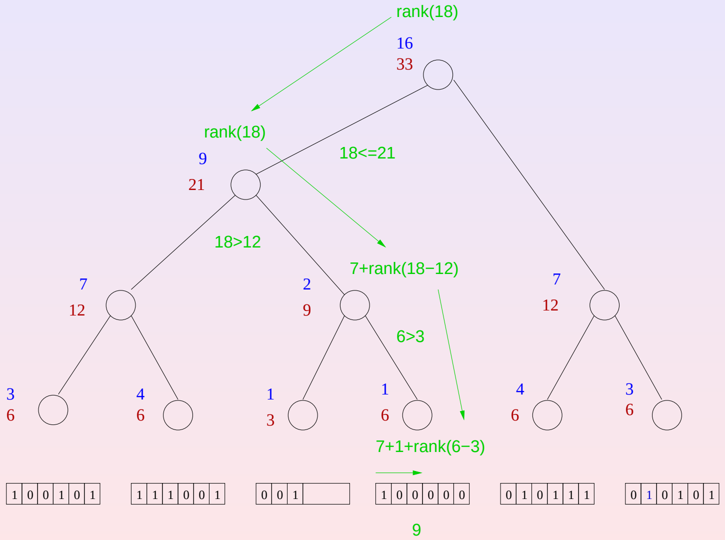
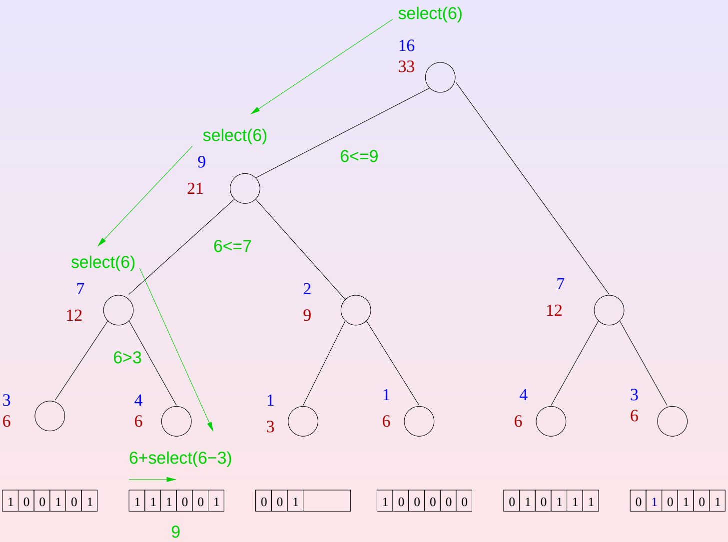

rank(18)
16
33
rank(18)
9
21
18<=21
18>12
7+rank(18−12)
7
12
2
9
7
12
6>3
3
6
4
6
1
3
1
6
4
6
3
6
7+1+rank(6−3)
1 0 0 1 0 1
1 1 1 0 0 1
0 0 1
1 0 0 0 0 0
0 1 0 1 1 1
0 1 0 1 0 1
9


select(6)
16
33
select(6)
9
21
6<=9
select(6)
7
12
6<=7
2
9
7
12
6>3
3
6
4
6
1
3
1
6
4
6
3
6
6+select(6−3)
1 0 0 1 0 1
1 1 1 0 0 1
0 0 1
1 0 0 0 0 0
0 1 0 1 1 1
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Insertions

- Find the position where the bit should be inserted.
- Insert it and move the following bits to the right.
- The bit that falls off is inserted into the next leaf.
- The process is repeated until...
- ... we reach a partial leaf, which is not totally full.
- Only one out of $f(n)$ leaves can be partial.
- Extra space is $n/f(n)$ bits.
- Insertion time is $O(\log n + f(n)^2)$.

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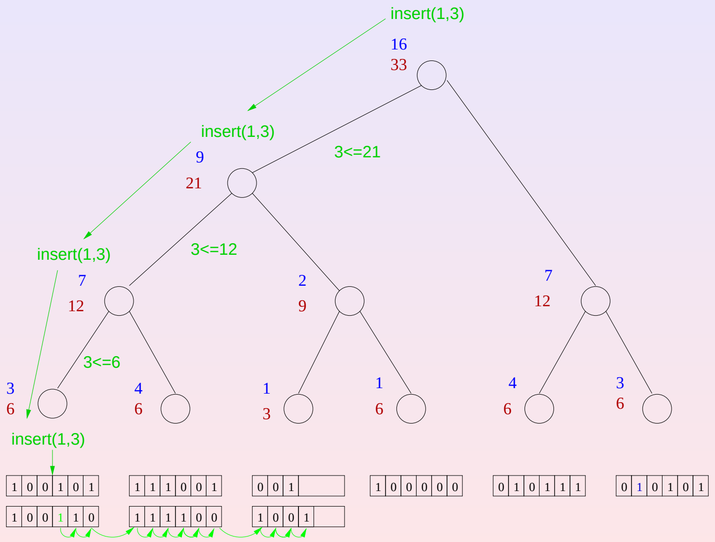
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Partial Leaves

- How to guarantee that there are at most $n/f(n)$ of them?
- Upon insertion, look at the next $2f(n)$ leaves.
- If a partial one is found, propagate insertion up to it.
- Otherwise, propagate $f(n)$ times and create a partial leaf.
- When a partial leaf gets filled, it is not called partial anymore.
- Deletion is similar.
- When a partial leaf gets empty, it must be removed.

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Tree Manipulation

- Each insertion/deletion requires at most one tree node insertion/deletion.
- But $O(f(n))$ subtree size/weight must be updated towards the root.
- Yet, those are contiguous, so there are $O(\log n + f(n))$ nodes involved.

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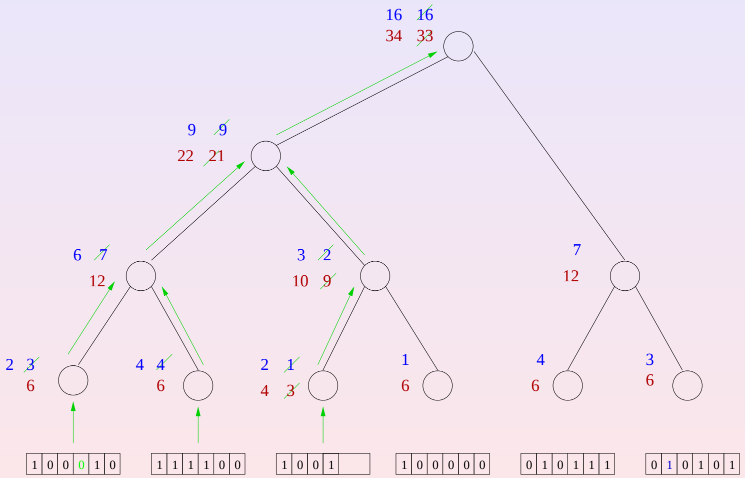
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Analysis

- Space: $n + O(n/f(n))$ bits.
- Time: $O(\log n + f(n)^2)$ at most.
- We choose $f(n) = \sqrt{\log n}$ to achieve
  - $n + o(n)$ bits of space
  - $O(\log n)$ time for all operations
- Changing $\log n$: Too technical, see paper.

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Identifier Encoding

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- B is divided into blocks of $b = (\log n)/2$ bits.
- Each block is represented as a pair (c, o).
  - c = number of bits set in the block.
  - o = index of the block within those of class c.
- Overall space is $nH_0 + o(n)$.
- Inspired in Raman, Raman, and Rao's static solution.

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- Overall space is $nH_0 + o(n)$.
- Inspired in Raman, Raman, and Rao's static solution.

Identifier Encoding

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- B is divided into blocks of $b = (\log n)/2$ bits.
- Each block is represented as a pair (c, o).
  - $c =$ number of bits set in the block.
  - $o =$ index of the block within those of class c.
- Overall space is $nH_0 + o(n)$.
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Operations

- Rank and select are again simple (with 4-Russian methods).
- A bit insertion can double the length of a block.
- Ex: 0000 1111 0000 1111 ...
  into 1000 0111 1000 0111 1....
- We can propagate up to $f(n)\log n$ bits to the next leaf.
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Operations

- On a red-black tree this requires $O(f(n))$ time plus recolorings.
- Just as recomputing subtree size/weights, recoloring costs $O(\log n + f(n))$.
- Change of $\log n$, etc. very similar.
- We get our final result:
  - $nH_0 + o(n)$ bits of space.
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Outline

Searchable partial sums with indels

The problem

- Sequence A of n numbers of k bits, $k = O(\log n)$.
- Sum: $\sum_{j=1}^{i} A_j$.
- Search: at which index the sum exceeds j.
- Insert and delete numbers.

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Our solution

Applying our simplest structure

- We have nk bits, handled in chunks of k bits.
- Internal tree nodes store number and sum of subtree chunks.
- Sum is solved like *rank*, Search like *select*.
- Insertion and deletion of chunks is totally analogous to bits.
- Easy to solve all operations in $O(\log n)$ time.

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Comparison to the existing solution

- The best current result by Hon, Sadakane and Sung, ISAAC 2003.
- It requires $kn + o(kn)$ bits of space.
- Solves queries in $O(\log_b n)$ time for $b = \Omega(\text{polylog}(n))$.
- But updates require $O(b)$ amortized time.
- b seems to be at least $\Theta((\log n / \log \log n)^2)$.
- Our query complexities are worse.
- But our update complexities are better, and worst-case.

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Wavelet trees

Structure

- By Grossi, Gupta, and Vitter, SODA 2003.
- Solves rank/select queries over a general sequence S.
- Let σ be the alphabet size.
- Balanced binary tree, dividing the alphabet into two at each node.
- Leaves correspond to individual symbols.
- Each node stores a bitmap telling which branch each position took.

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- A *rank* query over S is solved via $\log\sigma$ *rank*s over the bit vectors (top-down).
- Similarly with *select* (bottom-up).
- Symbol S_i can be discovered also with a top-down process.
- If the bit streams are represented using Raman et al's structure...
- ... the whole wavelet tree takes $nH_0(S) + o(n\log\sigma)$ bits of space...
- ... and it solves all operations in $O(\log\sigma)$ time.
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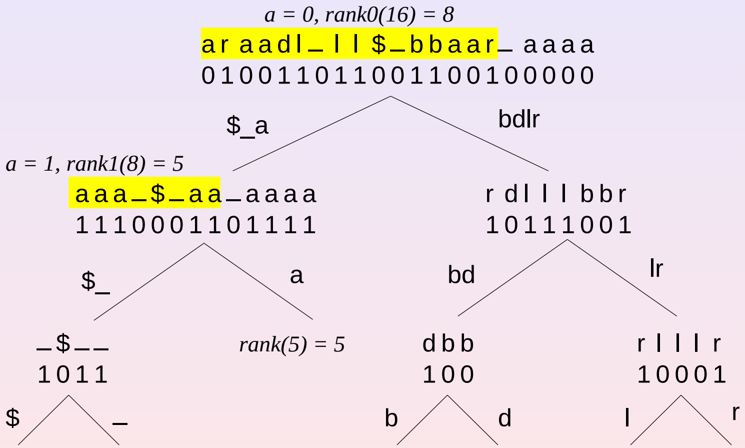
a = 0, rank0(16) = 8
ar aadl_ l l $_bbaar_ aaaa
0100110110011001000000
$_a
bdlr
a = 1, rank1(8) = 5
aaa_$_aa_aaaa
1110001101111
r dl l l bbr
10111001
$_
a
bd
lr
_$__
1011
rank(5) = 5
dbb
100
r l l l r
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$
_
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Wavelet trees

A dynamic version

- Use our dynamic solution for bit vectors instead.
- The operations now require $O(\log n \log \sigma)$ time.
- Within the same time we can insert and delete symbols from S.
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Outline

FM-indexes

The FM-index concept

- By Ferragina and Manzini, FOCS 2000
- Let T be a text (sequence) of length n.
- Let P be a pattern (sequence) of length $m << n$.
- Apply the Burrows-Wheeler Transform (a permutation) to T, $bwt(T)$.
- Answer $rank$ queries over $bwt(T)$.
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- To locate the occurrences: sample the suffix array.

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Text indexes

A simple realization

- A wavelet tree on $S = bwt(T)$.
- Space is $nH_0(T) + o(n \log \sigma)$.
- Time for counting is $O(m \log \sigma)$.
- But it is static.
- We would like to handle a collection of texts $\mathcal{C}$:
  - Insert a new text to $\mathcal{C}$.
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- By Chan, Hon, and Lam, CPM 2004.
- Two versions

Aspect	Version 1	Version 2
Space	$O(n\sigma)$	$O(n \log \sigma)$
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- Very recent result:
  The wavelet tree of $bwt(T)$, compressed with Raman et al.'s technique, takes actually $nH_h(T) + o(n \log \sigma)$ bits of space, for any $h \leq \alpha \log_\sigma n$, $\alpha < 1$.
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- Improved solutions to searchable partial sums with indels.
- First compressed structure for general sequences: Dynamic wavelet trees.
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