Statistical Encoding of Succinct Data Structures

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Outline

1. Background
   - Motivation
   - The k-th order empirical entropy
   - Statistical encoding
2. Entropy-bound succinct data structure
   - Idea
   - Data structures
   - Decoding Algorithm
   - Space requirement
   - Supporting appends
3. Application to full-text indexing
   - Succinct full-text self-indexes
   - The Burrows-Wheeler Transform
   - The wavelet tree

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Motivation

Previous work

- In recent work, Sadakane and Grossi [SODA'06] introduced a scheme to represent any sequence S using $nH_k(S) + O(\frac{n}{\log_\sigma n}((k+1)\log\sigma + \log\log n))$ bits of space.
- The representation permits us to extract any substring of size $\Theta(\log_\sigma n)$ in constant time, and thus it completely replaces S under the RAM model.
- This permits converting any succinct structure using $o(n\log\sigma)$ bits of space on top of S, into a compressed structure using $nH_k(S) + o(n\log\sigma)$ bits overall, for any $k = o(\log_\sigma n)$.

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- We extend previous works, by obtaining slightly better space complexity and the same time complexity using a simpler scheme based on statistical encoding.
- We show that the scheme supports appending symbols in constant amortized time.
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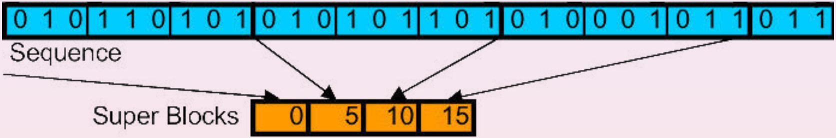
Example: a simple rank structure

Definition

$rank_1(S, i)$ = number of ones in $S[1 \ldots i]$.

Example: a simple rank structure



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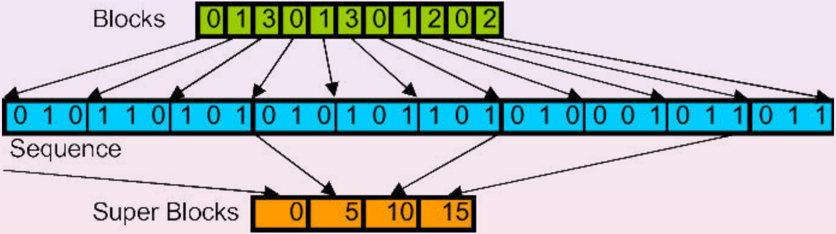
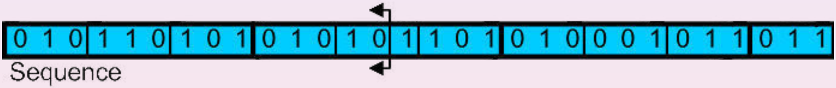


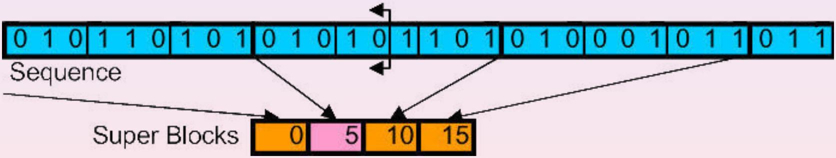
Example: a simple rank structure

rank(14)=



Example: a simple rank structure

rank(14)=5+



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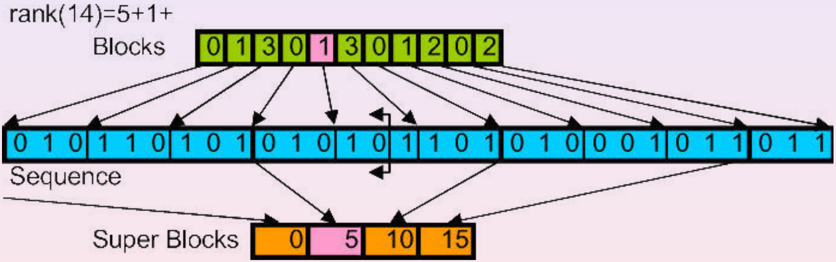

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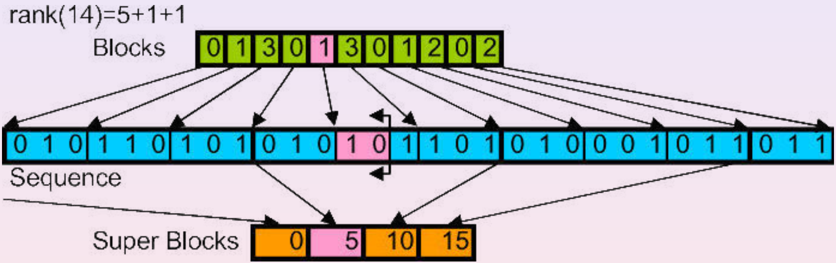

$$rank_1(S, 14) = 5 + 1 + 1.$$

Outline

The k-th order empirical entropy

Definition

- The empirical entropy is defined for any string S and can be used to measure the performance of compression algorithms without any assumption on the input.
- The k-th order empirical entropy captures the dependence of symbols upon their context. For $k \geq 0$, $nH_k(S)$ provides a lower bound to the output of any compressor that considers a context of size k to encode every symbol of S.

$$H_k(S) = \frac{1}{n} \sum_{w \in \Sigma^k} |w_S| H_0(w_S). \qquad (1)$$

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Semi-static Statistical encoding

Descriptions

- Given a k-th order modeler, which will yield the probabilities $p_1, p_2, \ldots, p_n$ for the symbols, we will encode the successive symbols of S trying to use $-\log p_i$ bits for s_i. If we reach exactly $-\log p_i$ bits, the overall number of bits produced will be $nH_k(S) + O(k \log n)$.
- Different encoders provide different approximations to the ideal $-\log p_i$ bits (Huffman coding, Arithmetic coding).
- Given a statistical encoder E and a semi-static modeler over sequence $S[1, n]$, we call $E(S)$ the bitwise output of E. We call $f_k(E, S)$ the extra space in bits needed to encode S using E, on top of $nH_k(S)$.

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Encoders

- Arithmetic coding essentially expresses S using a number in $[0,1)$ which lies within a range of size $P = p_1 \cdot p_2 \cdots p_n$. We need $-\log P = -\sum \log p_i$ bits to distinguish a number within that range (plus two extra bits for technical reasons).
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Entropy-bound succinct data structure

Idea

- Given a sequence $S[1, n]$ over an alphabet A of size σ, we encode S into a compressed data structure S' within entropy bounds. To perform all the original operations over S under the RAM model, it is enough to allow extracting any $b = \frac{1}{2} \log_\sigma n$ consecutive symbols of S, using S', in constant time.

Outline

Data structures

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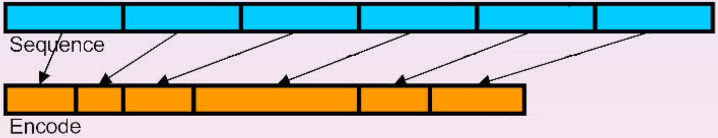
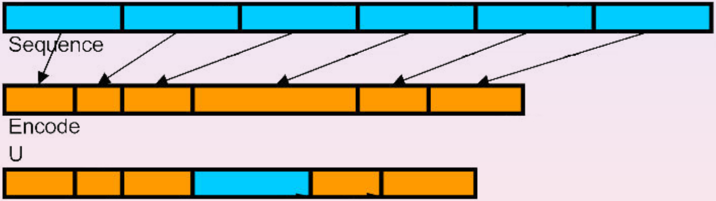


Data structures



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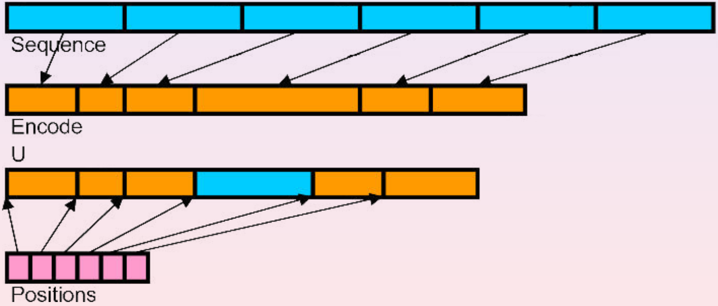

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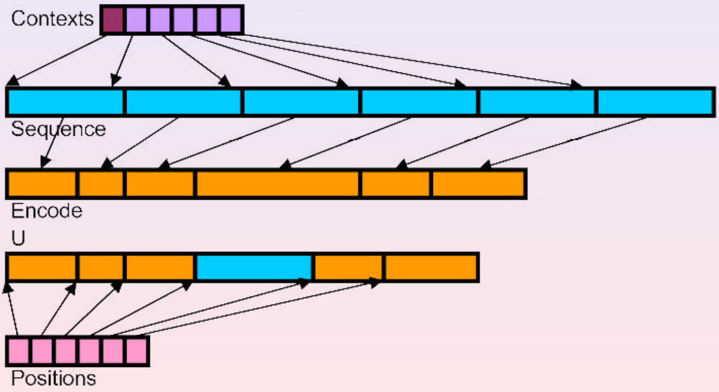

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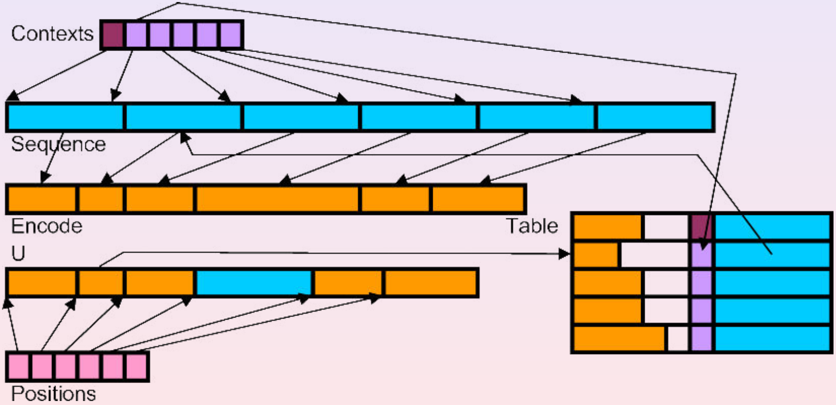

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Decoding Algorithm

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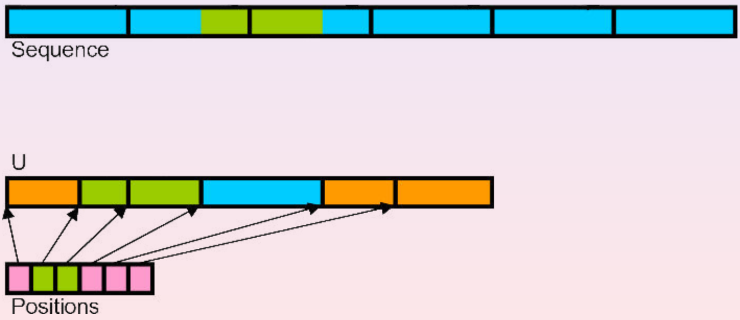

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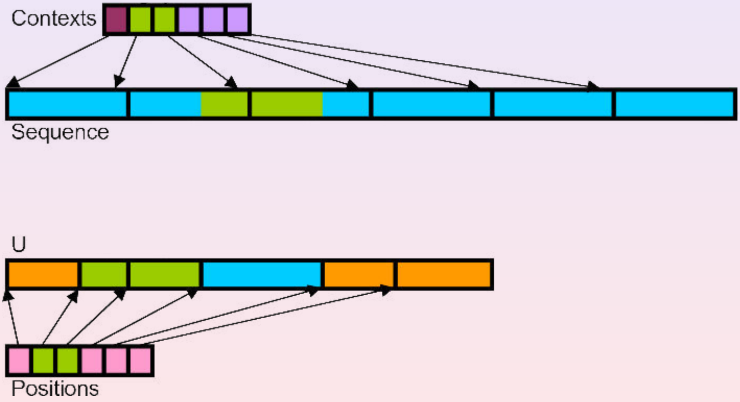

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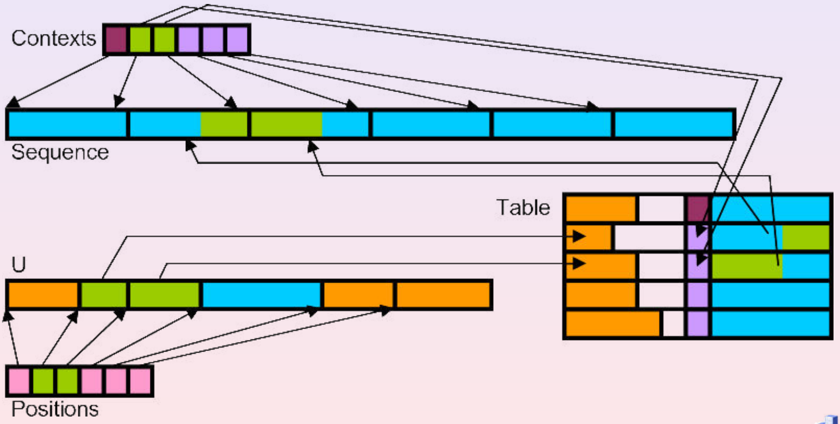

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Space requirement

Size of U

- $|U| = \leq \sum_{i=0}^{\lfloor n/b \rfloor} |E_i| = nH_k(S) + O(k \log n) + \sum_{i=0}^{\lfloor n/b \rfloor} f_k(E, S_i)$, which depends on the statistical encoder E used.
- Huffman: $f_k(\text{Huffman}, S_i) < b$, thus we achive $nH_k(S) + O(k \log n) + n$ bits.
- Arithmetic: $f_k(\text{Arithmetic}, S_i) \leq 2$, thus we achive $nH_k(S) + O(k \log n) + \frac{4n}{\log_\sigma n}$ bits.

Other structures

- Contexts: $(n/b)k \log \sigma = O(nk \log \sigma / \log_\sigma n)$
- Positions: $O(n \log \log n / \log_\sigma n)$
- Table: $\sigma^k \; n^{1/2} \log n / 2$

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Space requirement

Theorem

Let $S[1, n]$ be a sequence over an alphabet A of size σ. Our data structure uses $nH_k(S) + O(\frac{n}{\log_\sigma n}(k \log \sigma + \log\log n))$ bits of space for any $k < (1 - \epsilon)\log_\sigma n$ and any constant $0 < \epsilon < 1$, and it supports access to any substring of S of size $\Theta(\log_\sigma n)$ symbols in $O(1)$ time.

Corollary

Our structure takes space $nH_k(S) + o(n \log \sigma)$ if $k = o(\log_\sigma n)$.

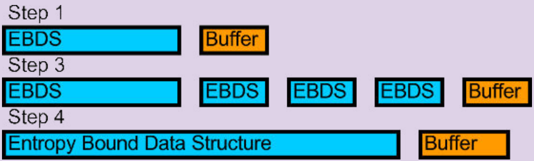
Outline

Supporting appends

Theorem

The structure supports appending symbols in constant amortized time and retains the same space and query time complexities.

Append scheme



Outline

Succinct full-text self-indexes

Definition

- A succinct full-text index is an index that uses space proportional to the compressed text. Those indexes that contain sufficient information to recreate the original text are known as self-indexes. Some examples are the FM-index family and the LZ-index.

Burrows-Wheeler Transform (BWT)

BWT

- The FM-index family is based on the Burrows-Wheeler Transform (BWT). The BWT of a text T, $T^{bwt} = bwt(T)$, is a reversible transformation from strings to strings, which is easier to compress by local optimization methods.
- An important property of the transformation is: if $T[k] = T^{bwt}[i]$, then $T[k-1] = T^{bwt}[LF(i)]$, where
  - $LF(i) = C[T^{bwt}[i]] + Occ(T^{bwt}[i], i)$.
  - $C[c]$ is the total number of text characters which are alphabetically smaller than c.
  - $Occ(c, i)$ is the number of occurrences of character c in the prefix $T^{bwt}[1, i]$.
- This property permits navigating the text T backwards.

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Succinct full-text self-indexes

Wavelet tree

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- The wavelet tree $wt(S)$ built on S is a binary tree, built on the alphabet symbols, such that the root represents the whole alphabet and each node has the information telling which of its characters belongs to the left/right child.

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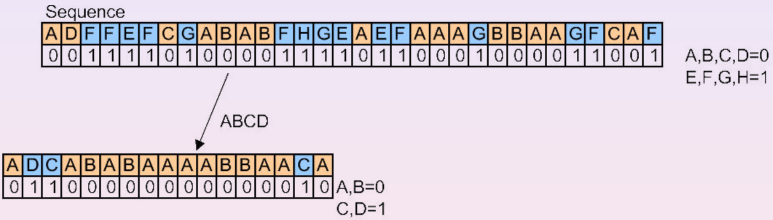
Wavelet tree

Sequence

A	D	F	F	E	F	C	G	A	B	A	B	F	H	G	E	A	E	F	A	A	A	G	B	B	A	A	G	F	C	A	F
0	0	1	1	1	1	0	1	0	0	0	0	1	1	1	1	0	1	1	0	0	0	1	0	0	0	0	1	1	0	0	1

A,B,C,D=0
E,F,G,H=1

Wavelet tree



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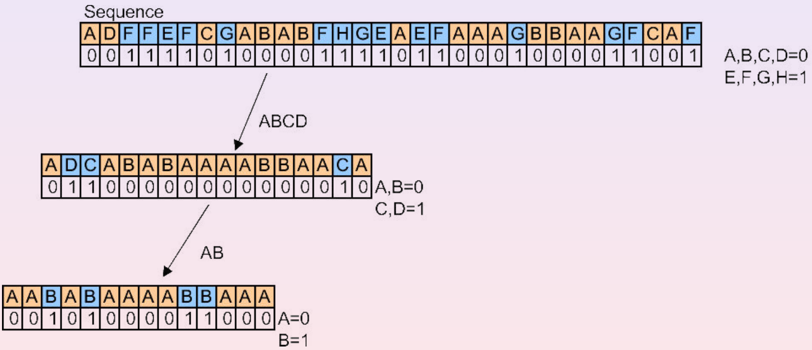

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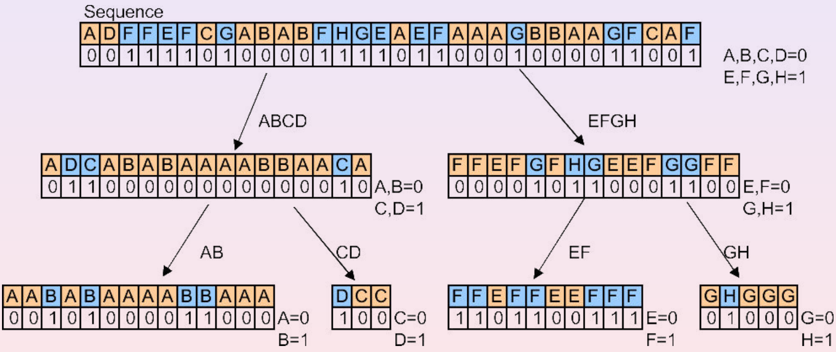

Outline

Relationship between T^{bwt} and T

We could encode $S = bwt(T)$ within $nH_k(S) + o(n \log \sigma)$ bits, but how this relates to $nH_k(T)$?

Lemma

Let $S = bwt(T)$, where $T[1, n]$ is a text over an alphabet of size σ. Then $H_1(S) \leq 1 + H_k(T) \log \sigma + o(1)$ for any $k < (1 - \epsilon) \log_\sigma n$ and any constant $0 < \epsilon < 1$.

Relationship between T^{bwt} and T

Application

- We can get at least the same results of the Run-Length FM-Index by compressing $bwt(T)$ using our structure.
- We can implement the original FM-index ($5nHk(T) + O(n\sigma \log\log n / \log_\sigma n + (\sigma/e)^{\sigma+3/2} n^\gamma \log_\sigma n \log\log n)$ bits) using $nH_k(T)\log\sigma + n + o(n)$ bits.

Outline

Relationship between $wt(S)$ and S

- $wt(S)$ takes $nH_0 + o(n \log \sigma)$ bits of space and permits answering *Occ* queries in time $O(\log \sigma)$
- Many FM-index variants build on the wavelet tree:
  - SSA takes $nH_0 + o(n \log \sigma)$ bits of space
  - RLFM-index takes $nH_k \log \sigma + o(n \log \sigma)$
  - AF-FM-index takes $nH_k + o(n \log \sigma)$
- In all cases the bitmaps of the $wt(S)$ are compressed to their H_0, but we can now compress them to H_k.

- Is k-th order entropy preserved across a wavelet tree? (it is for $k = 0$)

Lemma

The ratio between $H_k(wt(S))$ and $H_k(S)$, can be at least $\Omega(\log k)$. More precisely, $H_k(wt(S))/H_k(S)$ can be $\Omega(\log k)$ and $H_k(S)/H_k(wt(S))$ can be $\Omega(n/(k \log n))$.

Consequence

Applying our structure over the bitmaps of the wavelet tree does not perfectly translate into $nH_k(S)$ overall space, as there is a penalty factor of at least k in the worst case. But in the best, it can be much better than $nH_k(S)$.

Summary

- We presented a scheme based on k-th order modeling plus statistical encoding to convert any succinct data structure on sequences into a compressed data structure.
- This simplifies and slightly improves previous work.
- We presented a scheme to append symbols to the original sequence within the same space complexity and with constant amortized cost per appended symbol.
- We found relationships between the entropies of two fundamental structures used for compressed text indexing.

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Future work

- Making our structure fully dynamic
- Better understanding how the entropies evolve upon transformations such *bwt* or *wt*.
- Testing our structure in practice.
- Currently working on another way to solve the same problem. That would permit full dynamism using recent work (see next talk).

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Thank you!!