Constrained Tree Inclusion

14th Annual Symposium on Combinatorial Pattern Matching

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Remark *The slides got shuffled as the speaker stumbled over a slope when approaching the podium*

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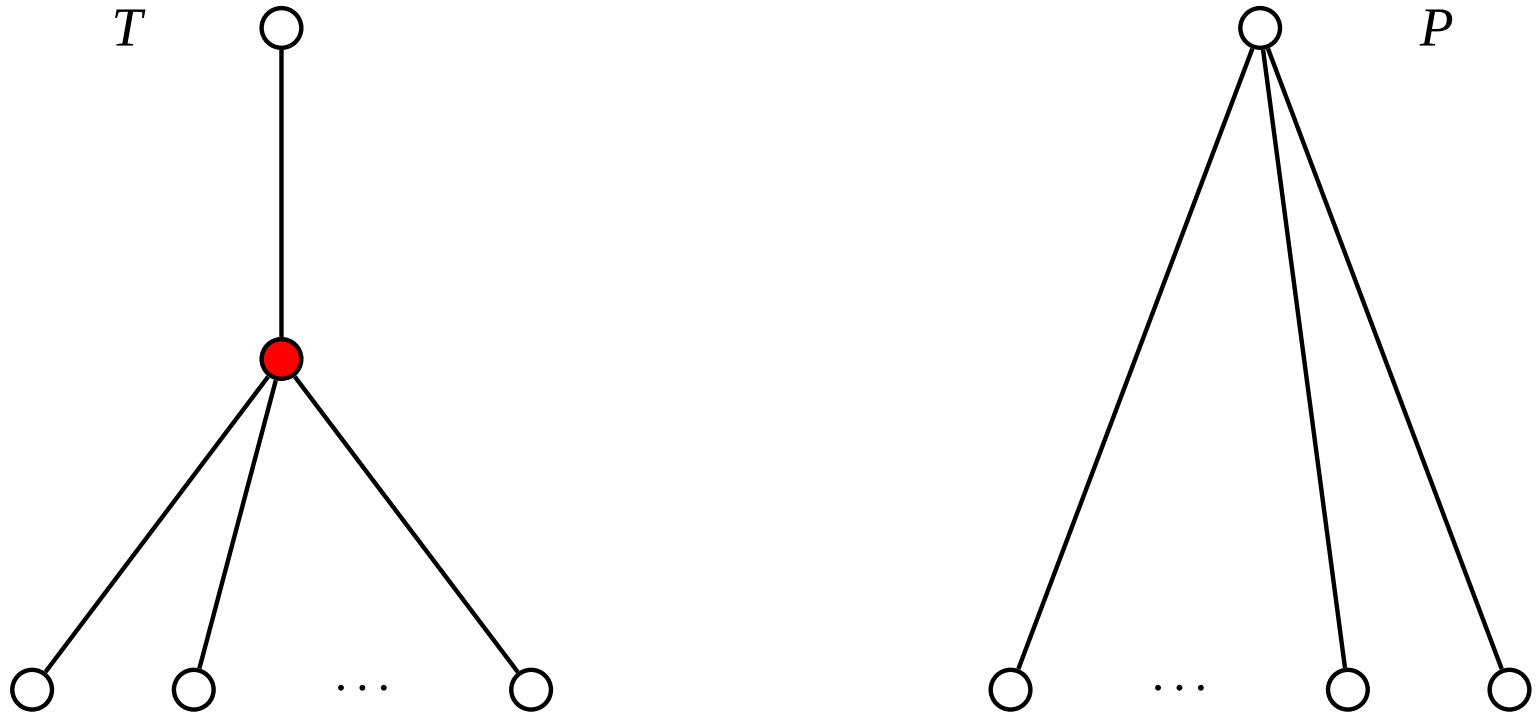

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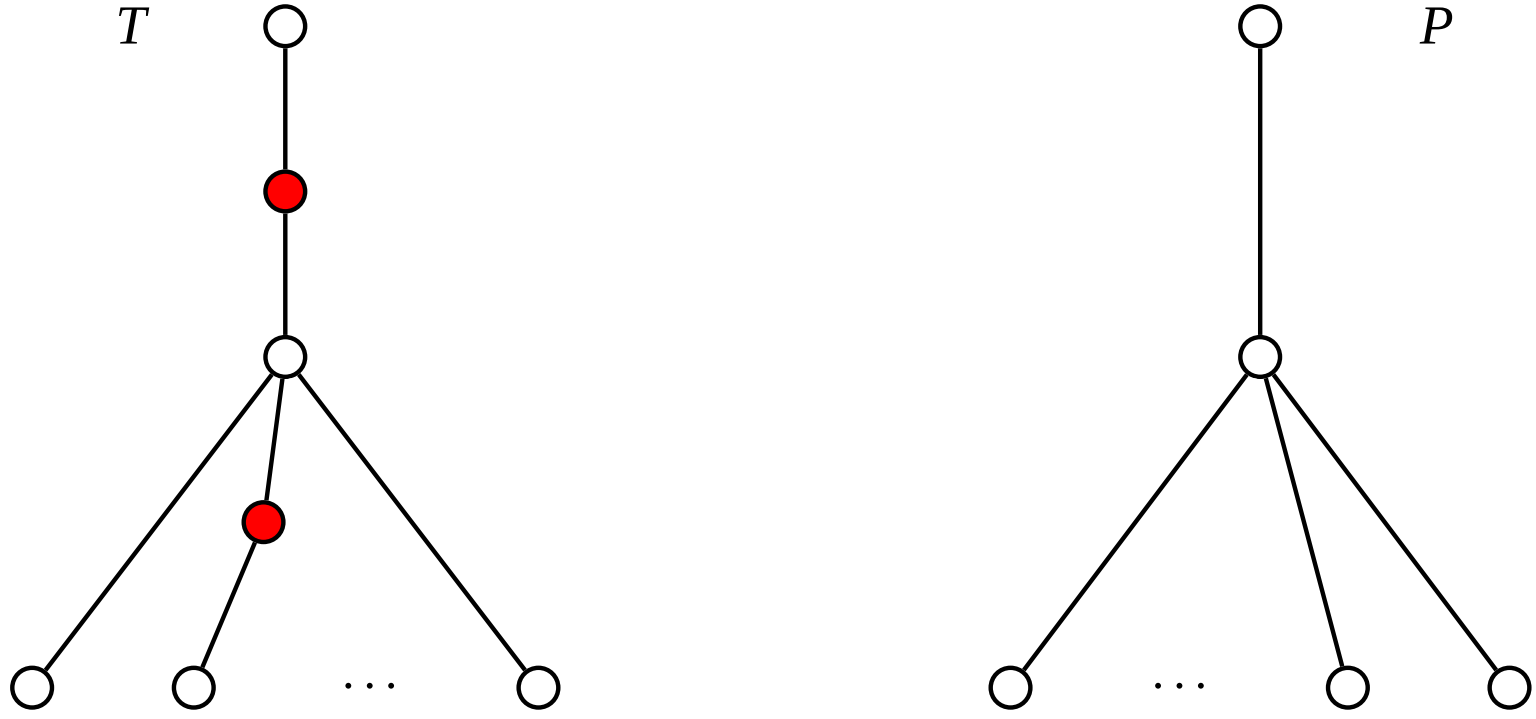

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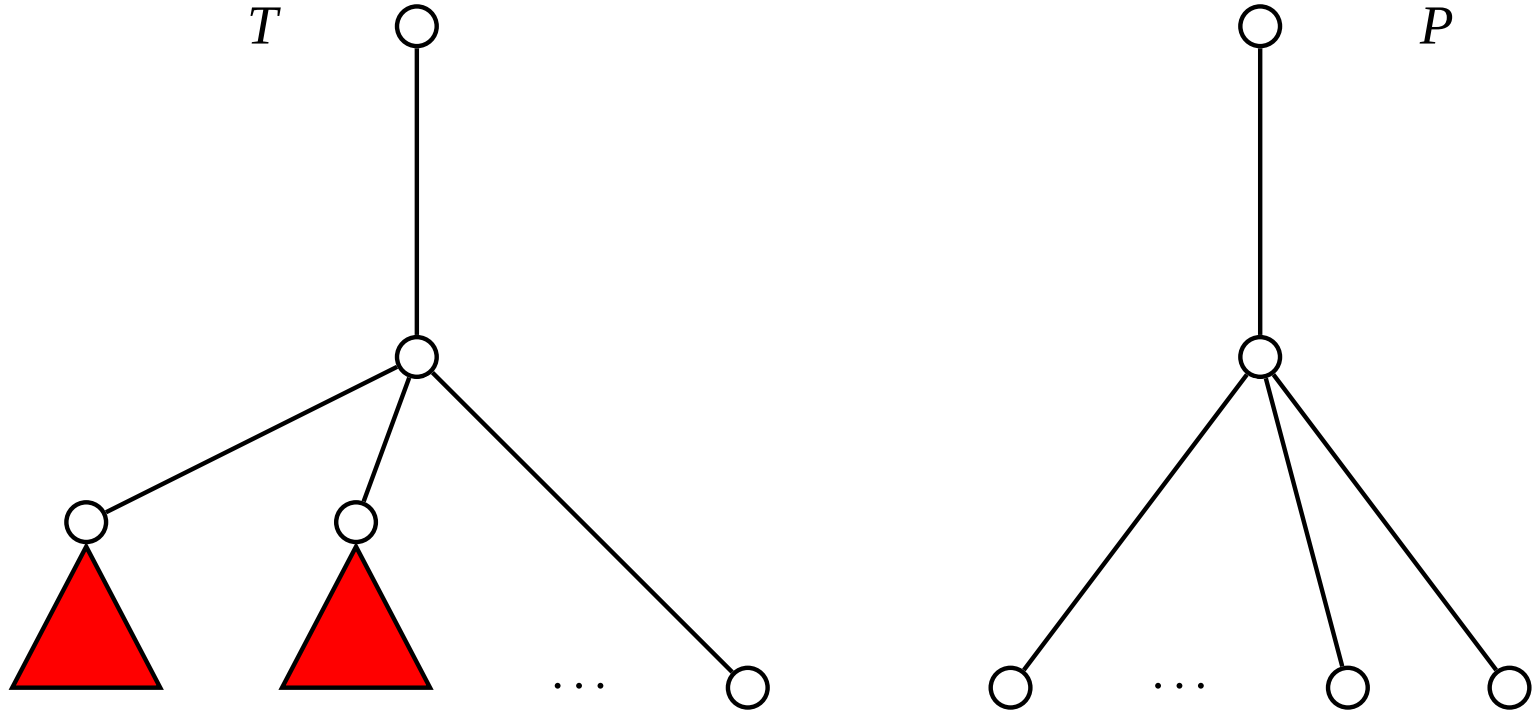

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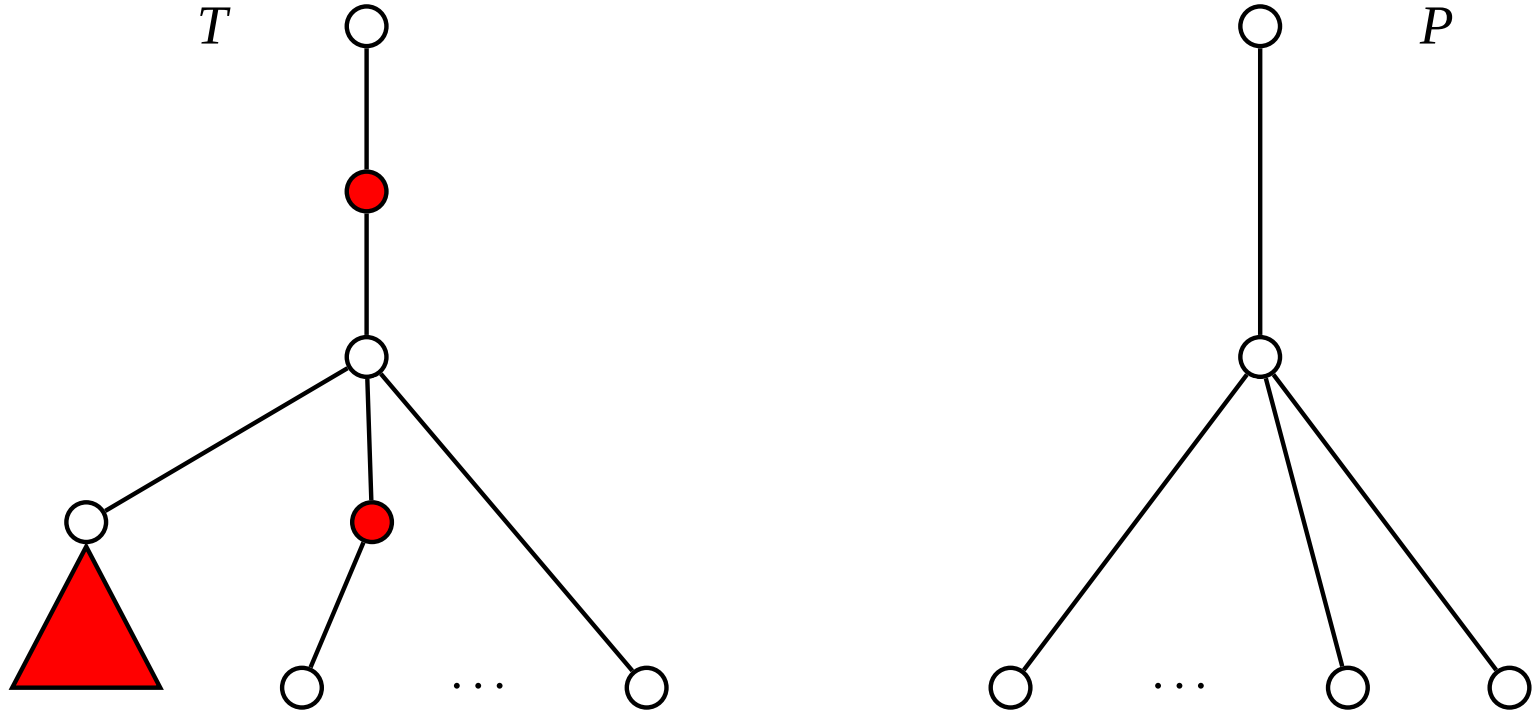

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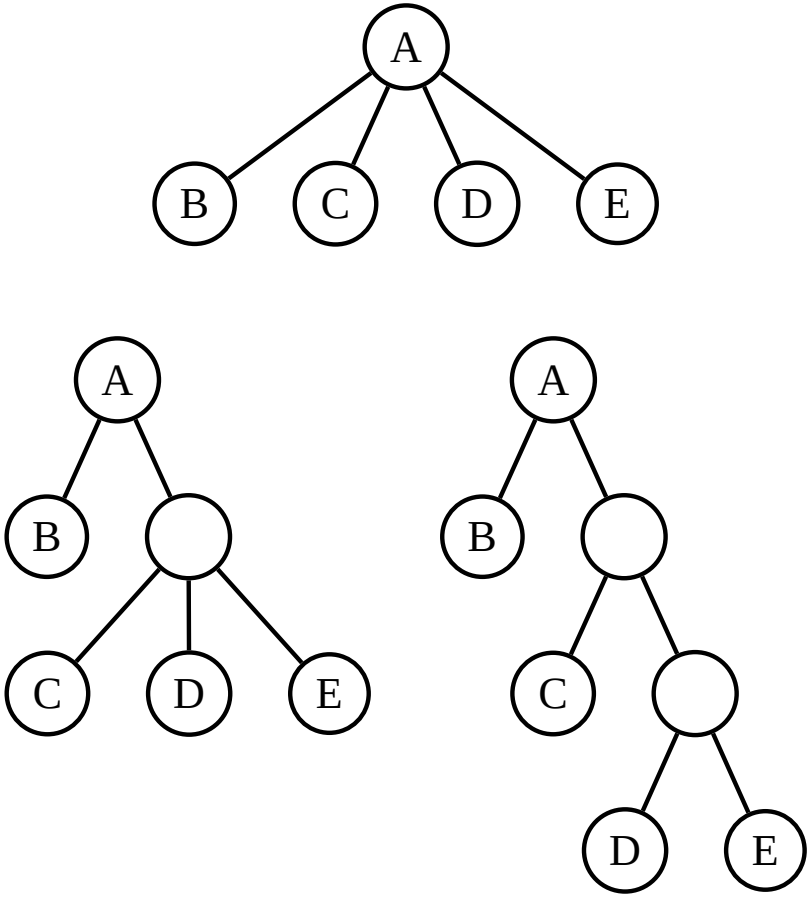
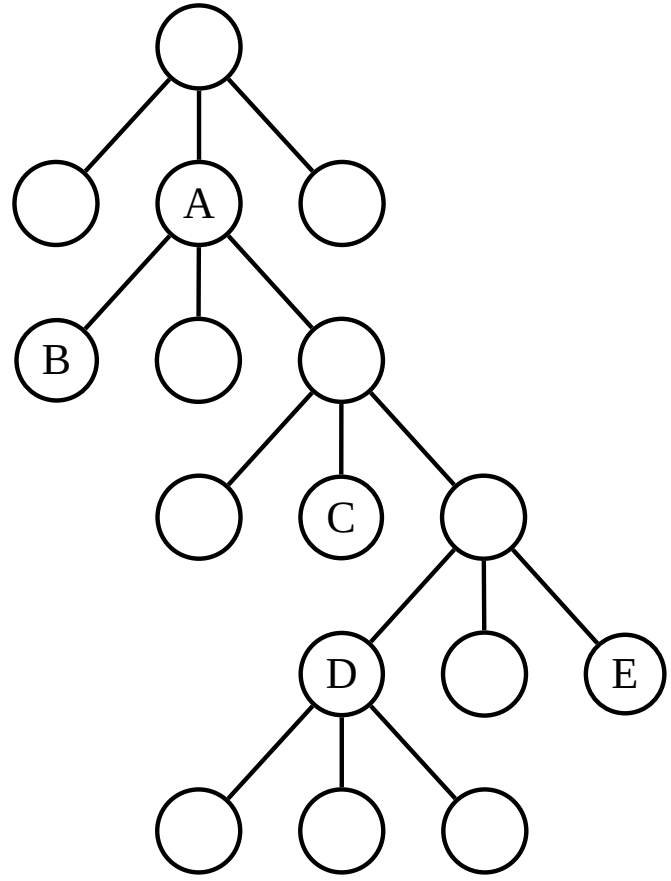
    - Solvable for ordered trees by dynamic programming in $O(mn)$ time and space, in the worst case and also on the average
- These drawbacks stem from the generality of tree inclusion

Motivation

- Three forms of the same query are all included at the node labeled A in the text tree, shown to the right of the picture





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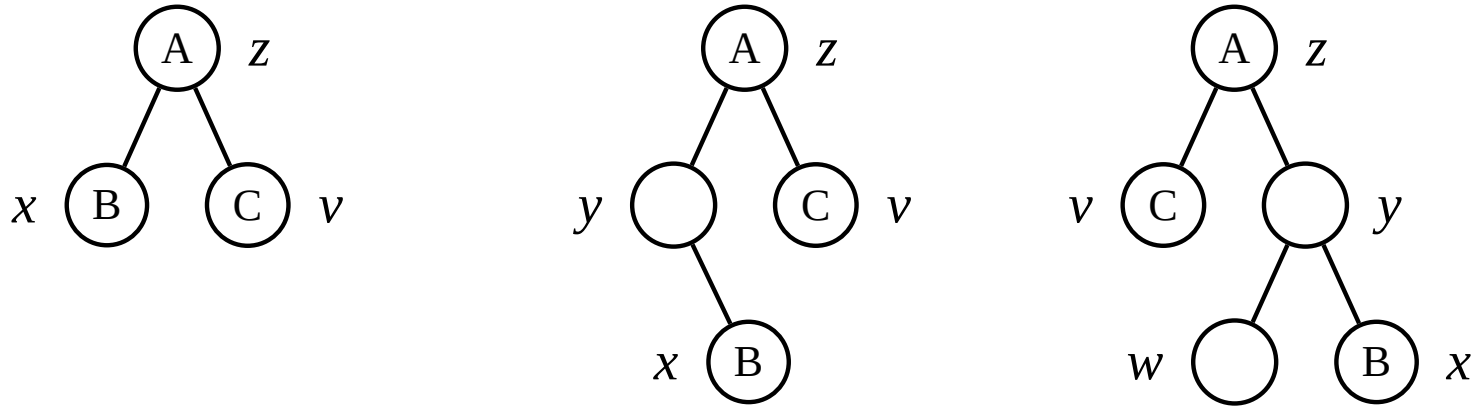
$P = T_2 \cong delete(T_1, y)$ $\quad$ $T_1 \cong delete(T_0, w)$ $\quad$ $T_0 = T$



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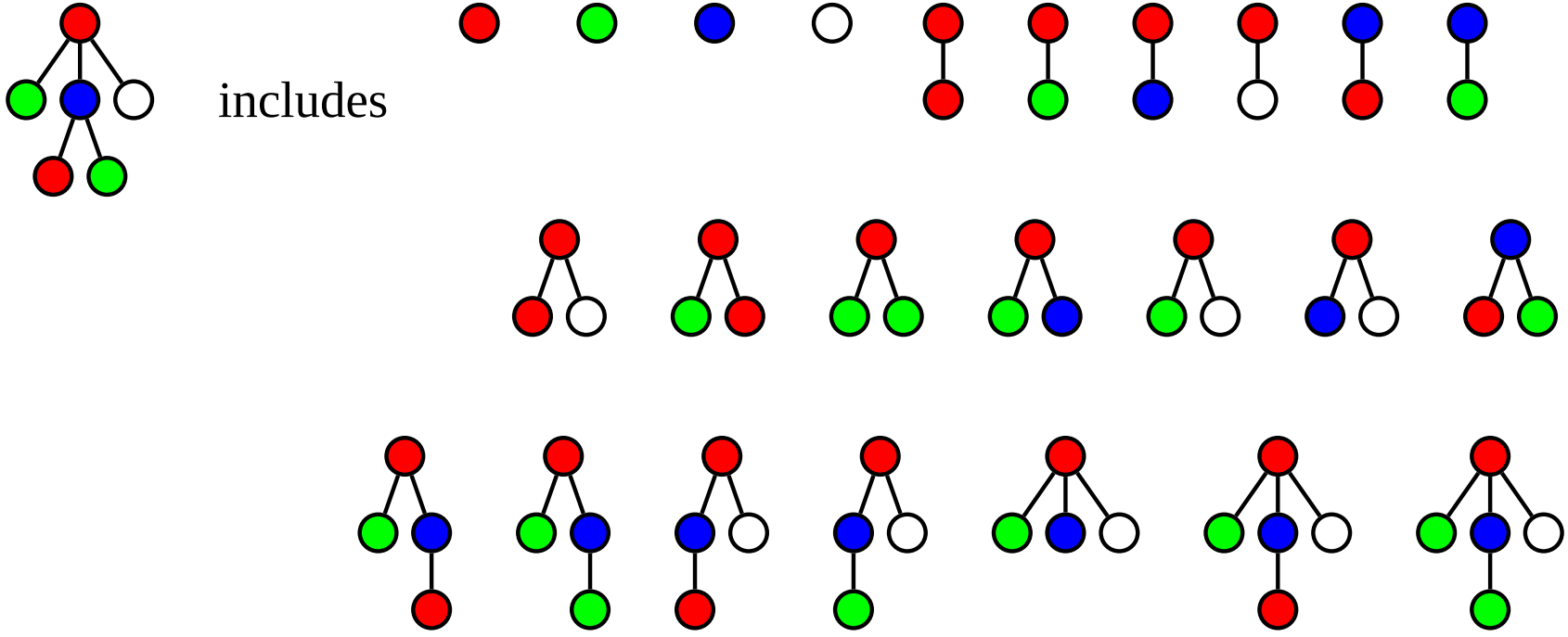

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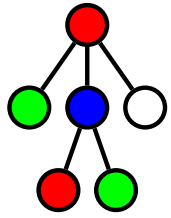

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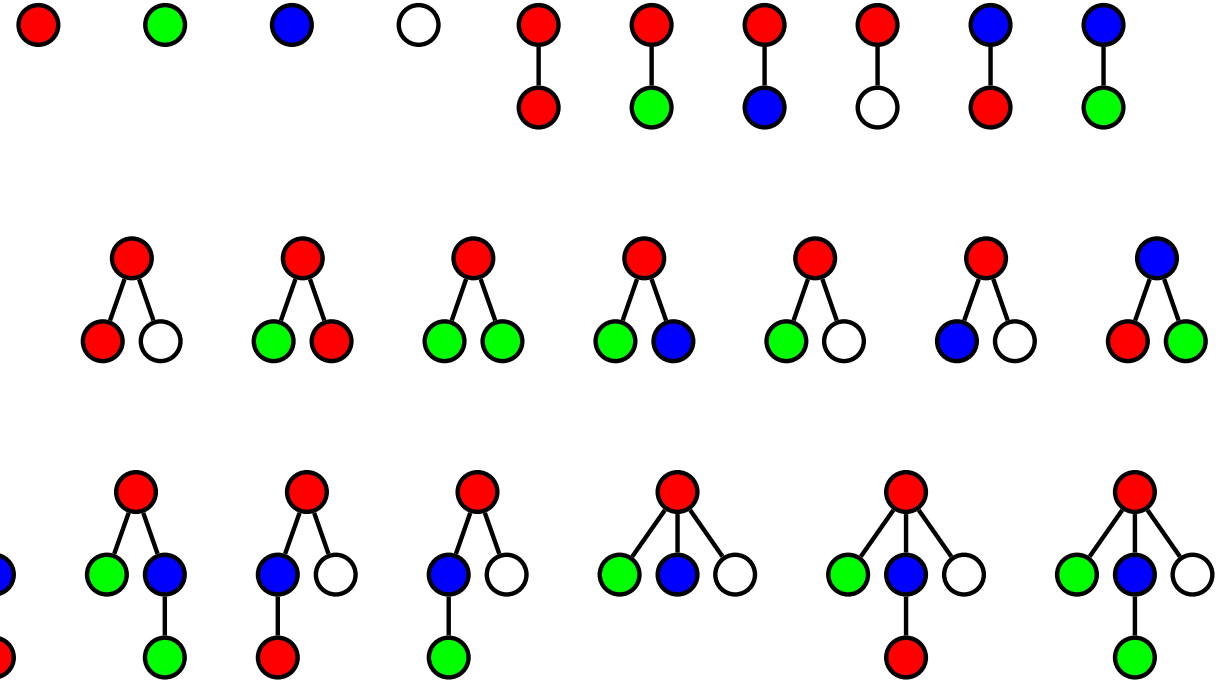
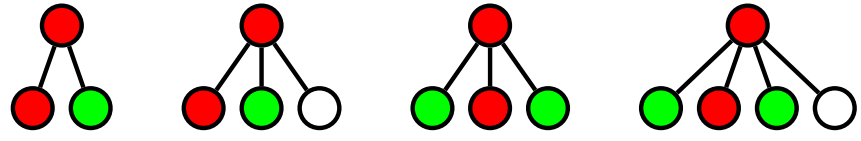


and does not include



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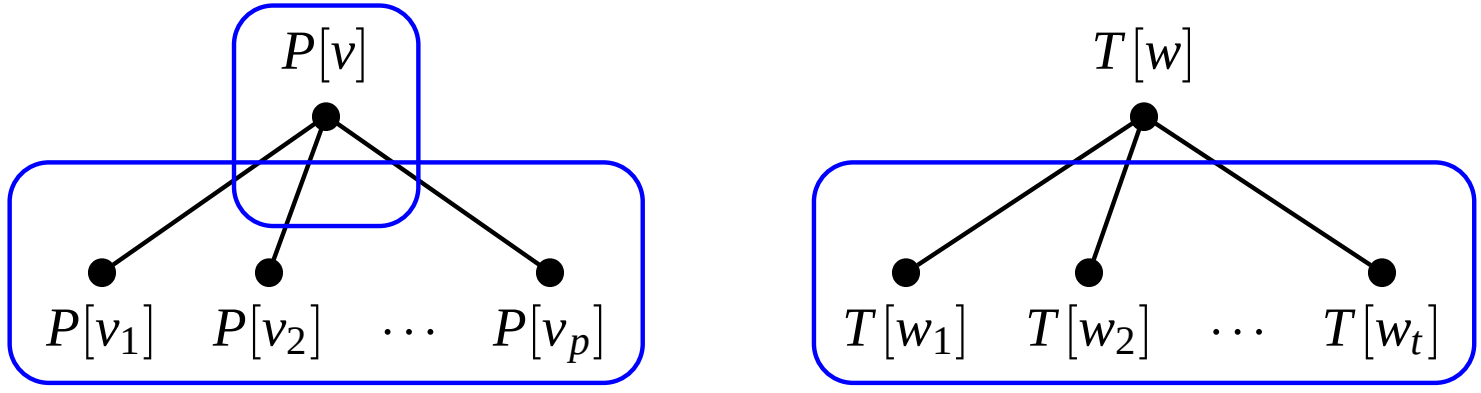

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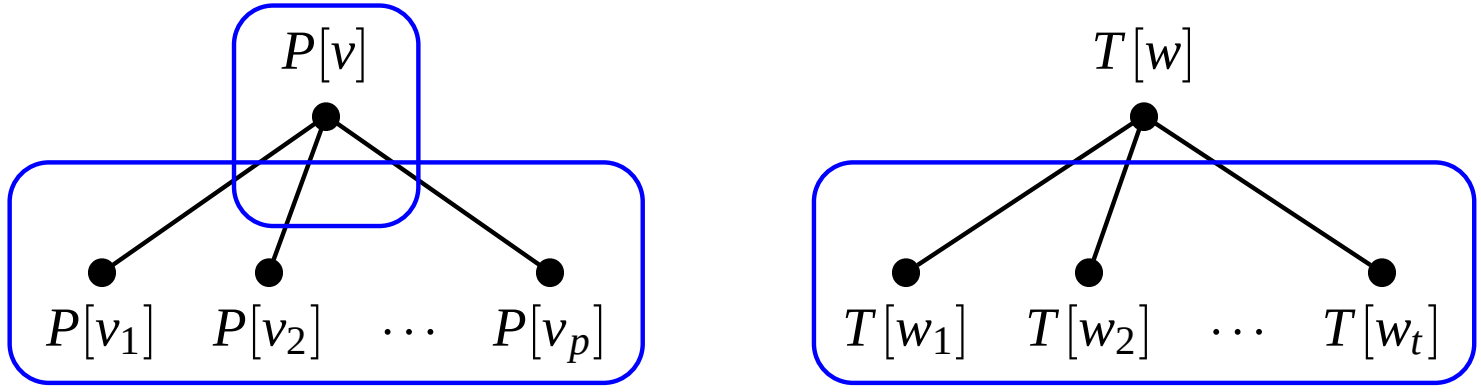

- That is, it suffices to know if $P[x] \sqsubseteq T[y]$ for all $x \in \{v, v_1, v_2, \ldots, v_p\}$ and $y \in \{w_1, w_2, \ldots, w_t\}$

Extension for Ordered Trees

- An ordered bipartite graph is a bipartite graph $G = (V \cup W, E)$ with orderings $V = (v_1, v_2, \ldots, v_p)$ and $W = (w_1, w_2, \ldots, w_q)$

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- Noncrossing bipartite matching is equivalent to sequence inclusion

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$$\sum_{i=1}^{n}\sum_{j=1}^{m} O(outdeg(w_i)outdeg(v_j)) = \sum_{i=1}^{n} O(m \cdot outdeg(w_i)) = O(mn)$$

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  By deleting all nonroot nodes of T in postorder, the children (if any) of a node will have already been deleted when the node is considered for deletion, meaning the node has become a degree-one node (a leaf), which can thus be deleted

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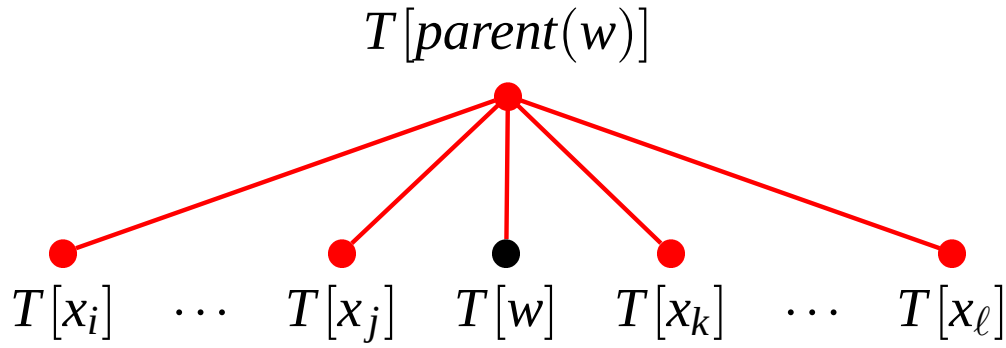

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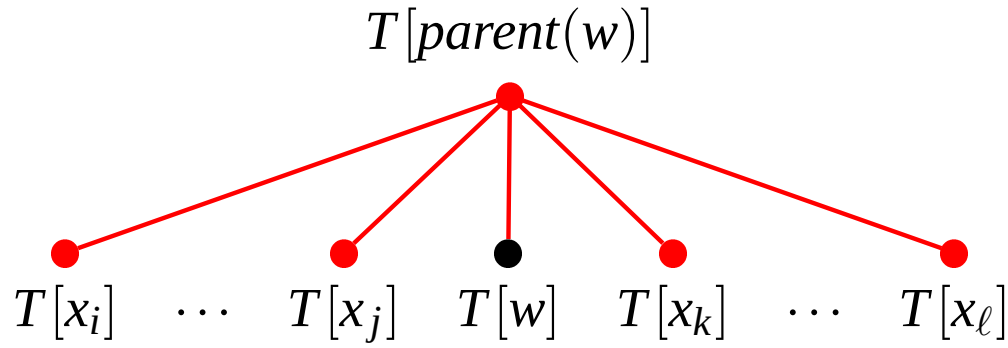

$P[v] \sqsubseteq T[w]$, and $T[w]$ can be obtained from $T[parent(w)]$ by deleting $T[x]$ for all siblings x of node w and, then, deleting node $parent(w)$, which has become either a degree-one or a degree-two node

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- Let $v \in V(P)$ have children $v_1, v_2, \ldots, v_p$, and let $w \in V(T)$ have children $w_1, w_2, \ldots, w_t$. Then, $P[v] \sqsubseteq T[w]$ if and only if either there is a child w_j of w such that $P[v] \sqsubseteq T[w_j]$, or $label(v) = label(w)$ and there is a subset of p different nodes $\{u_1, u_2, \ldots, u_p\} \subseteq \{w_1, w_2, \ldots, w_t\}$ such that $P[v_i] \sqsubseteq T[u_i]$ for $1 \leqslant i \leqslant p$

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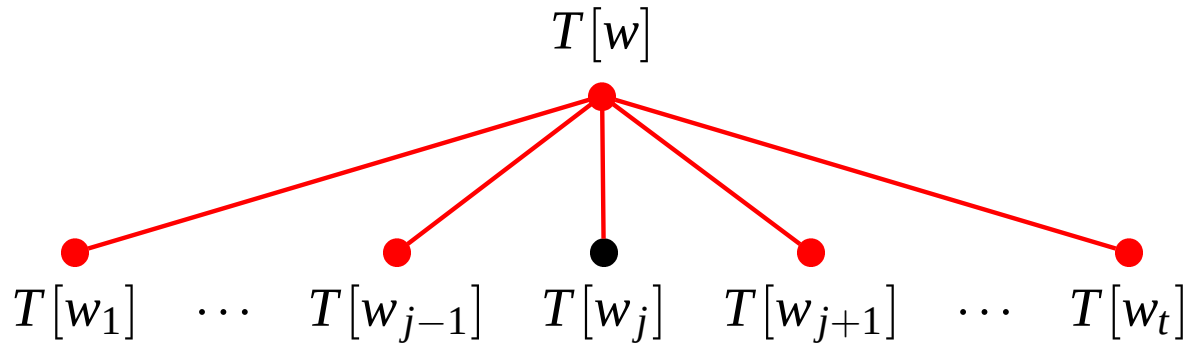

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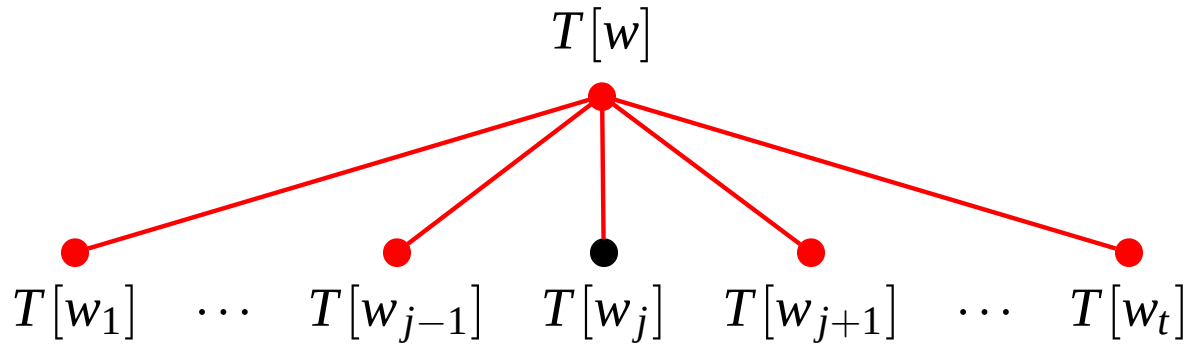

In the first case, $P[v]$ can be obtained from $T[w]$ by deleting $T[w_1], T[w_2], \ldots, T[w_{j-1}], T[w_{j+1}], \ldots, T[w_t]$ and, then, deleting node w, which has become either a degree-one or a degree-two node

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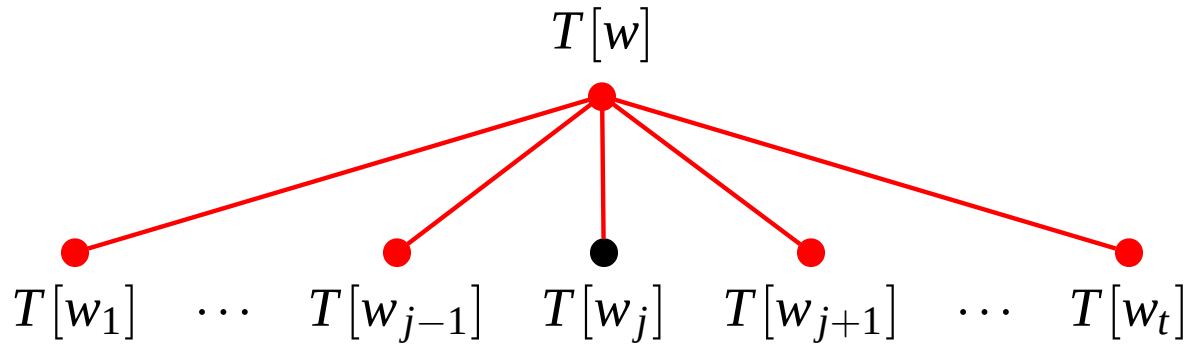

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In the second case, $P[v]$ can be obtained from $T[w]$ by deleting $T[w_i]$ for all $w_i \in \{w_1, w_2, \ldots, w_t\} \setminus \{u_1, u_2, \ldots, u_p\}$

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$$\sum_{i=1}^{n}\sum_{j=1}^{m} O(outdeg(w_i)outdeg(v_j)^{1.5})$$

$$= \sum_{i=1}^{n} O(m^{1.5}outdeg(w_i)) \quad (\text{because } \sum_{j=1}^{m} outdeg(v_j) = m-1)$$

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